# Modeling Acorns

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## 1 The task

In recent years there have been many applications of phase change materials (PCM) for latent heat storage [9]. In support of the experiments conducted at HUN-REN,[1] the need arose for a PCM container with specific qualities: (i) volume of around 200–300ml, (ii) high heat conductivity, (iii) easy construction with injection molding, (iv) good packing characteristics, and (iv) a somewhat Hungarian symbolism.

In view of (iv)–(v) we chose the shape of an acorn, as it is assumed to be naturally well-packable (see Fig. 1), and, while not distinctly Hungarian, it is something all Hungarians can relate to (it is also a suit in Tell-patterned playing cards widely used in the region).

Figure 1: A pile of acorns.

Heat conduction is enhanced by adding ridges to the body (Section 3.1), while requirement (iii) is achieved by splitting the model near the bottom rim of the 'cap' or *cupule* part (Section 3.2).

The report is structured as follows. In Section 2 we show our basic model, to which we add enhancements in Section 3. We have considered several alternatives, some of which are shown in Section 4. Finally, we discuss some characteristics of the proposed shape in Section 5.

[1]`https://www.ttk.hun-ren.hu/`

## 2 Initial model

Our initial model was a simple rotational surface. Its profile curve consists of two parametric curves (Fig. 2): a quadratic B-spline curve of two segments (red), with knot vector $\{0, 0, 0, 0.9, 1, 1, 1\}$, and a quintic Bézier curve (blue).

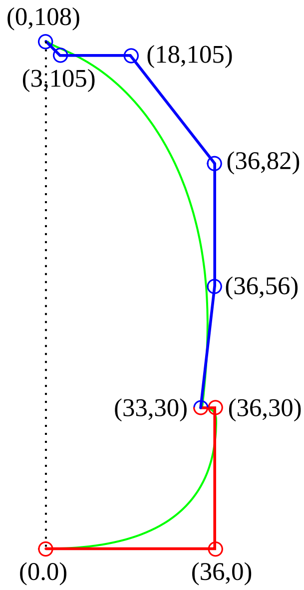


Figure 2: Profile control polygons (in millimeters).

Rotating this profile around the $y$ axis results in the basic shape shown in Figure 3. This was originally modeled in OpenSCAD, an open-source solid modeling language.[2] Here a stem—a 15mm long cylinder with a radius of 5mm—was also added to the bottom, and an offsetted model was subtracted to achieve a wall thickness of 1mm. The hole in the stem penetrates into the main body, as this will serve as the opening for injecting the phase change material into the container.

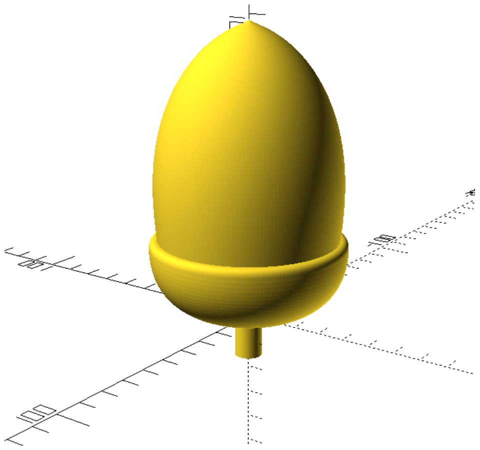

Figure 3: Base model in OpenSCAD.

# 3 Enhancements

The above model is aesthetically pleasing, but we could boost heat conductivity by increasing the surface area. Also, the container, as defined above, can be fabricated by 3D printing technology (e.g. by selective laser sintering (SLS), where internal supports are not needed), but not with injection molding. We take care of these problems below.

## 3.1 Adding ridges

A simple way to increase surface area is to add ridges to the object, such as a sinusoidal wave. This would also help maintining the flow of the surrounding liquid. The ridges should be shallower where the profile is closer to the $y$ axis, and also we decided that these features should smoothly vanish near the rim of the cupule. We have modified the control points of both curves in Figure 2 by multiplying the $x$ coordinates with

$$1 + 0.08 \cdot \sin(15\varphi) \cdot \alpha(y), \qquad (1)$$

where $\varphi$ is the rotational angle, $\alpha(y) = 1 - y/30$ for the cupule, and $\alpha(y) = (y - 30)/78$ for the *pericarp*. The result is shown in Figure 4.

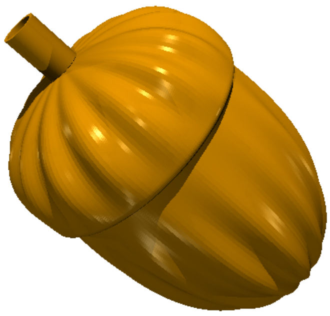

Figure 4: Container with ridged surface.

## 3.2 Splitting

Next we needed to split the model into two halves. A natural splitting point would be the meeting point of the cupule and the pericarp, but for the welding to be more robust, we chose to split 1mm closer to the stem, thereby leaving a larger contact area—see Figure 5 for a split view of the model (inner parts are shown in magenta). Note that we also incorporated a small ledge near the inner ring of the cupule to help with the assembly.

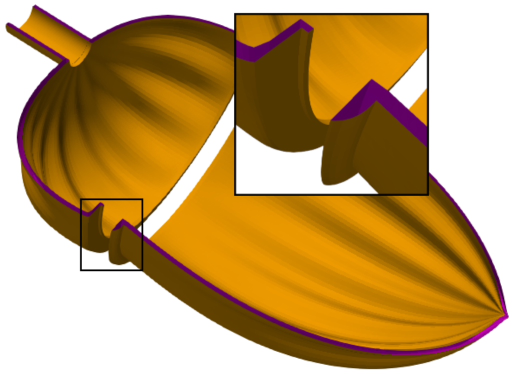

Figure 5: Split view of the split container.

[2] `https://openscad.org/`

# 4 Alternatives

The proposed model has many parameters. Some of the most salient are:

- No ridges on the cupule
- Non-vanishing ridges
- Twisted ridges
- Shorter/longer pericarp length

Examples are shown in Figure 6.

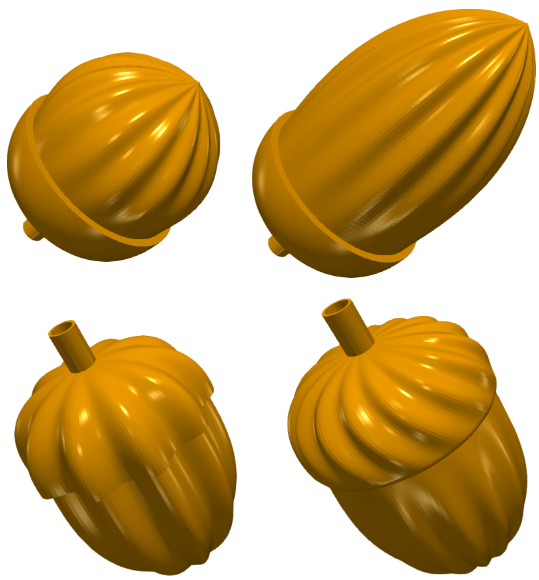

Figure 6: Alternative models.

# 5 Analysis

In this section we discuss some properties of the container: its volume and its probable packing ratio.

## 5.1 Volume

The volume is easily computed by the shoelace formula [1] applied to the inner (offsetted) mesh of the model (without the stem, so this is a closed mesh). The result is 273ml, which is inside the required range (the 'short' ang 'long' alternatives shown above exemplify the two extremes of the valid range, i.e., 200ml and 300ml, respectively).

## 5.2 Packing

The *packing problem*, i.e., how many of a given object can fit in a container of fixed size, is very hard, even for simple two-dimensional shapes [7].

An often-used approximation is to use a bounding ellipse. This is easily found by a derivative-free optimization, such as Powell's method [5], minimizing the area while changing the coordinates of the foci. (The semi-major axis can be computed by taking half of the maximum of the summed distances to the foci from all points of the silhouette.) The best fit is an ellipse of eccentricity 0.772, see Figure 7.

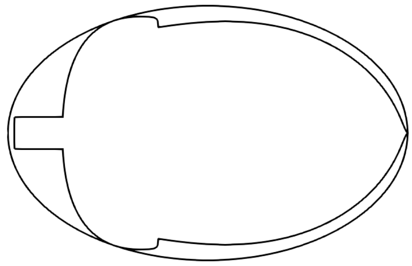

Figure 7: Best fit ellipse.

In 3D, we can use a spheroid (an ellipsoid where two of the axes are the same). Its volume can be computed by the familiar formula

$$V = \frac{4}{3}\pi ab^2, \tag{2}$$

where $a$ is the semi-major, and $b$ is the semi-minor axis. Using the volume of the *outer* shell, straightforward computation shows that the model fills 71.4% of the spheroid.

Finding an optimal packing of ellipses [8] or ellipsoids [4], even in a rectangular box, is still very hard. It is also not very useful: it would not tell us the probable packing ratio when our containers are put into actual use.

We can create a randomized simulation, where (in 2D) we only need to take care so that the ellipses do not intersect. This can be checked by looking at the roots of a cubic characteristic polynomial [2]. Similar results for ellipsoids are also available [10]. Such random packings fill only around 50% of the box, see Figure 8.

What if we take a regular packing, putting each spheroid into its axis-aligned bounding box? With

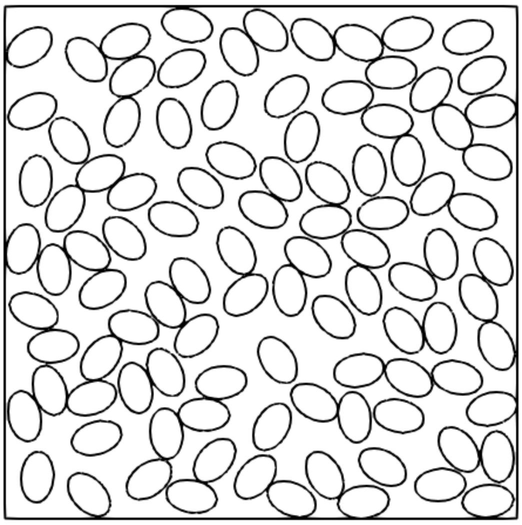

Figure 8: Randomized packing of ellipses.

a naïve packing, we get

$$\frac{\frac{4}{3}\pi ab^2}{8ab^2} = \frac{\pi}{6} \approx 52.4\%. \quad (3)$$

However, we can shift the second layer in a way that the spheroids rest on the 'shoulders' of the ones in the first layer, thus decreasing layer depth, see Figure 9. With this, we get a packing ratio of around 65%.

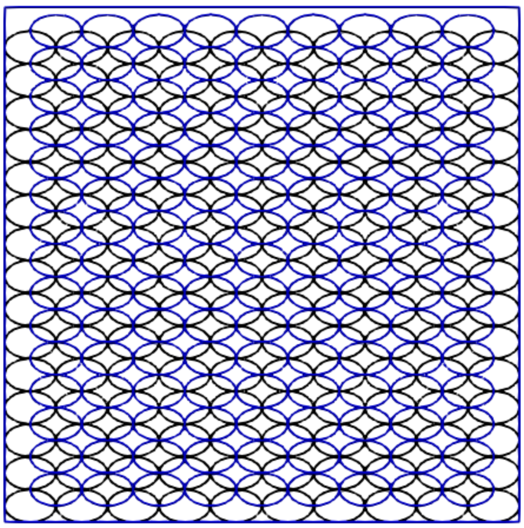

Figure 9: Two layers of spheroids.

As an experiment, we tried packing *Big Hit* milk chocolate peanuts in a *Fushimi cube* [3], folded from a 15cm origami paper, so its edge size is $150/\sqrt{13} \approx$ 41.6mm [6], and its volume is around 72ml.

The eccentricity of the peanuts, with 95% confidence interval, is $0.624 \pm 0.11$, which is slightly more rounded than our model. The mean semi-major axes are 9.45mm and 7.34mm, respectively. During the experiment, we could fit 19 of these into the box, leading to an occupied volume of approximately 41ml (57% packing ratio).

Figure 10: Origami box filled with chocolate-coated peanuts.

From the above we can assume that even with a non-regular packing we can take up at least 40% of the box volume with our containers.

## Conclusion and future work

We have shown the design process of our Acorn PCM container, and we discussed some of its properties, as well as a few alternative design decisions. With this, the work has just started. In the next time period, we plan to:

1. Fabricate a few test models with SLS printing
2. Conduct experiments and modify the model accordingly
3. Create a mold for mass production

## Acknowledgments

Unless otherwise noted, all 3D images were generated with the scientific visualization software package ParaView.[3]

[3] `https://www.paraview.org/`

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