A circular parameterization for multi-sided patches

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Why genuine multi-sided patches?
Parameterization
Domain evolution
Parallel tangents – periodic boundaries?

- Handled as multiple ribbons (CD)
- Handled as a single ribbon (GBS)
- Harmonic parameterization $\Rightarrow$ discrete solution
- High computation cost, not suitable for some patch types
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REJECTED
Preliminaries
Properties I – Height parameter basics [linear map]

1. $h = 0$ on the base side.
2. $h$ is continuous and varies monotonically.
3. $h$ changes uniformly from 0 to 1 on the sides adjacent to the base side.
4. $h \leq 1$ everywhere inside the domain.
5. $h = 1$ on all distant sides. [full mapping]
6. \( h'_{i-1} = -h'_{i+1} \) on the \( i \)th side. [constrained mapping]
All properties?

- Constrained Wachspress coordinates
- 1D multi-sided patch
- Singular blending function
All properties?

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Inverse map

- Unit circle with equal arcs
- Base: $[-\frac{\pi}{n}, \frac{\pi}{n}]$
- $\varphi = \frac{(2h+1)\pi}{n}$
- $\theta = h\pi \Rightarrow \theta : 0 \rightarrow \pi$
- $\psi = \theta - \varphi$
- $O = \left(\frac{\sin \theta}{\sin \psi}, 0\right)$
- $r = \left|\frac{\sin \varphi}{\sin \psi}\right|$
Examples
Constrained property & corner parameterization
Algorithm

- Line at $\hat{h} = \frac{1}{n-2} \Rightarrow \hat{u} = \cos \frac{\pi}{n-2}$
- Same circle for $h = 0$ and $h = 1$
- Idea: bisection search

```python
if u > \hat{u} then
    return bisection(\Delta, 0, \hat{h} - \varepsilon)
if u < \hat{u} then
    return bisection(\Delta, \hat{h} + \varepsilon, 1)
return \hat{h}
```

$$\Delta(h) = \|p - O(h)\| - r(h)$$
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Application
Overlap–GB patch

- Corner-based variation of the GB patch
- Needs a full, constrained parameterization

\[
S = \sum_{i=1}^{n} \sum_{j=0}^{\lfloor d/2 \rfloor} \sum_{k=0}^{\lfloor d/2 \rfloor} P_{ijk} B_j^d(h_{i+1}) B_k^d(h_i) + P_0 B_0,
\]
Conclusions

- Circle as multi-sided domain
- Height parameterization
  - Circular arcs
  - Full
  - Constrained
  - Efficient
- Overlap–GB patch
- Suitable for periodic boundaries
- $G^2$ cap for subdivision surfaces?
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Any questions?