A circular parameterization for multi-sided patches

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Outline

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Circular domain
- Inverse problem
- Examples
- Algorithm

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- Overlap–GB patch

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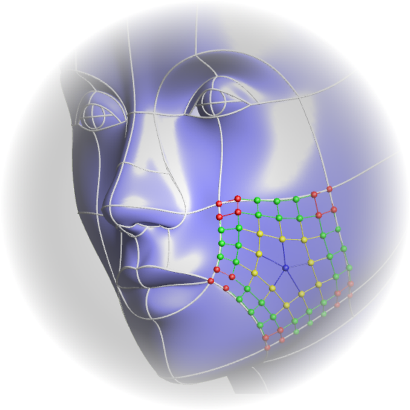
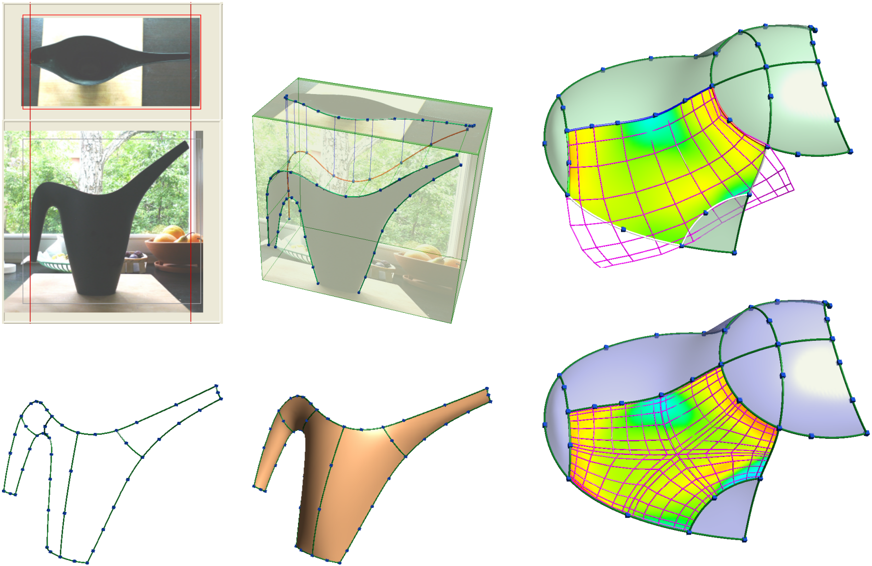
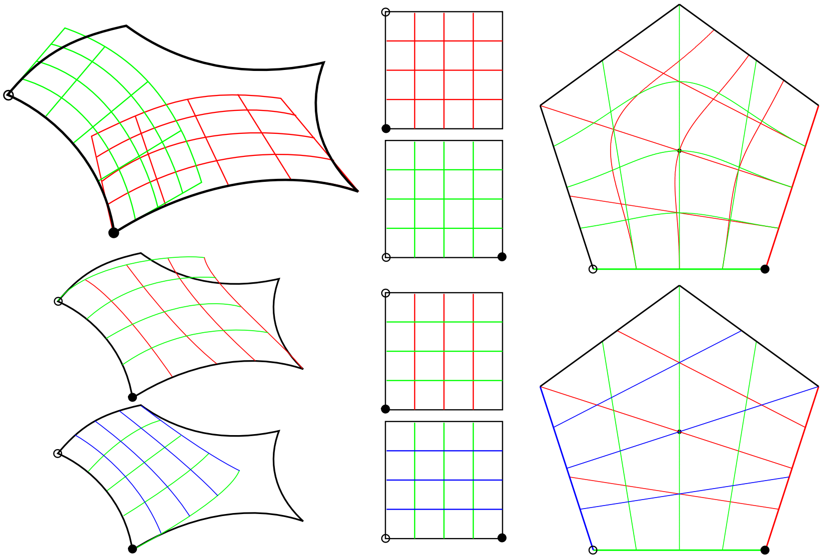


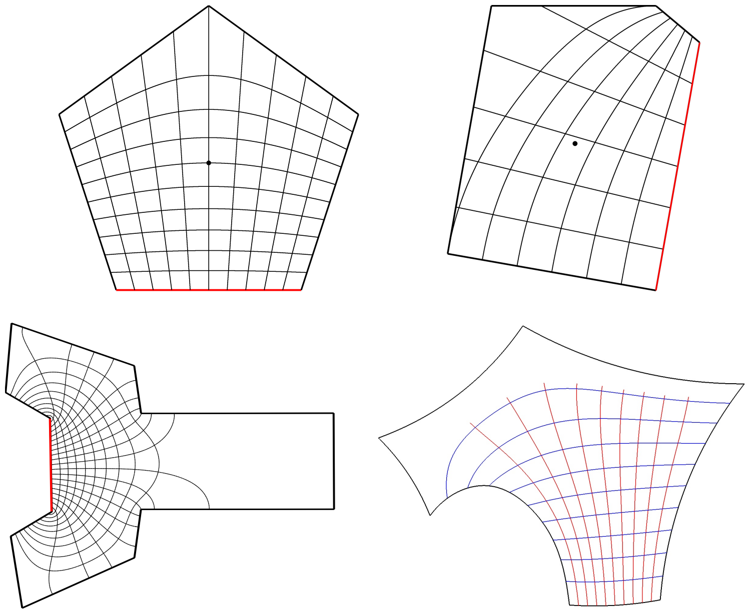
Why genuine multi-sided patches?



Parameterization

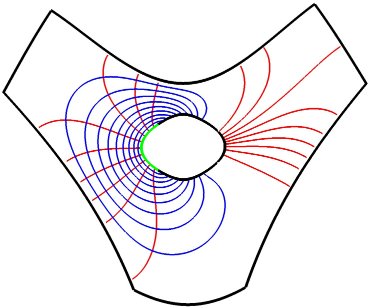


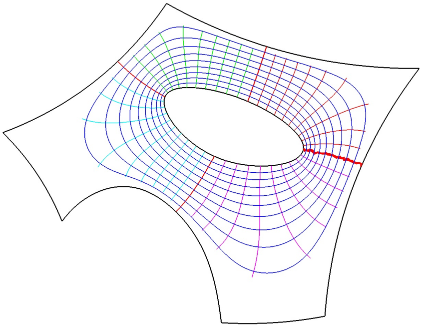
Domain evolution



Parallel tangents – periodic boundaries?

- Handled as multiple ribbons (CD)
- Handled as a single ribbon (GBS)
- Harmonic parameterization ⇒ discrete solution
- High computation cost, not suitable for some patch types





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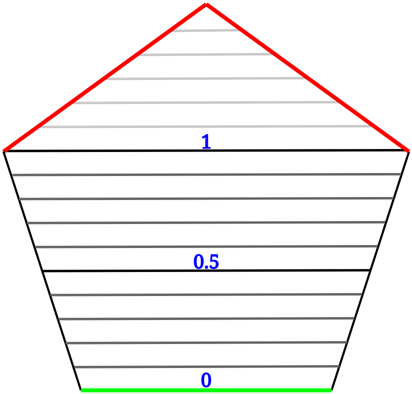
REJECTED

Preliminaries

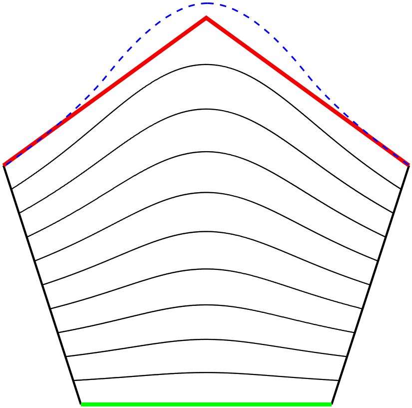
Properties I – Height parameter basics [linear map]

1. $h = 0$ on the base side.
2. h is continuous and varies monotonically.
3. h changes uniformly from 0 to 1
   on the sides adjacent to the base side.



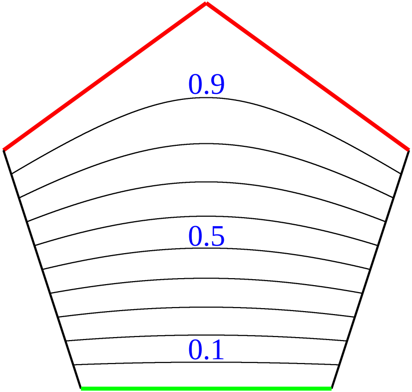
Properties II – Limited [interconnected map]

4. $h \leq 1$ everywhere inside the domain.



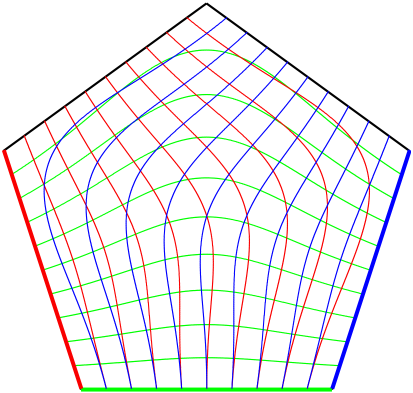
Properties III – Full [Wachspress map]

5. $h = 1$ on all distant sides. [*full mapping*]



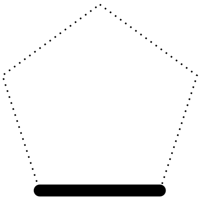
Properties IV – Constrained [interconnected map]

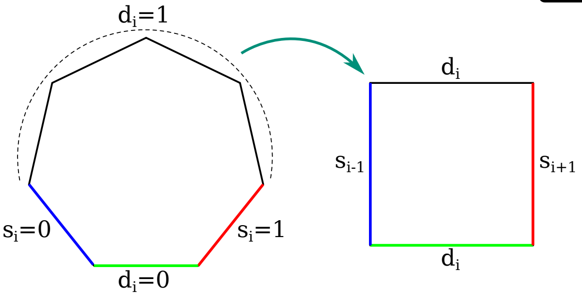
6. $h'_{i-1} = -h'_{i+1}$ on the ith side. [*constrained mapping*]



All properties?

- Constrained Wachspress coordinates
- 1D multi-sided patch
- Singular blending function





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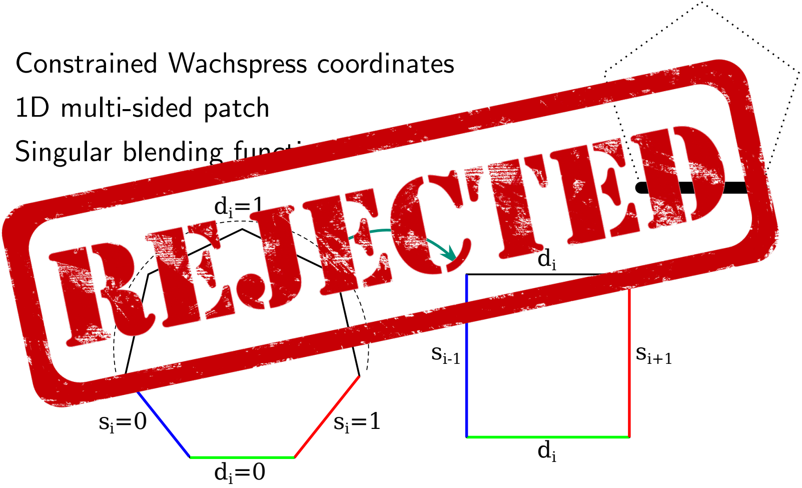
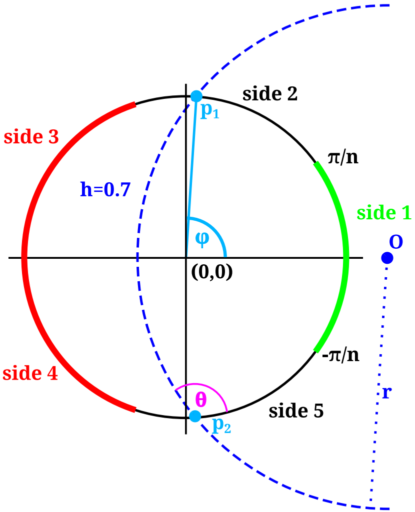


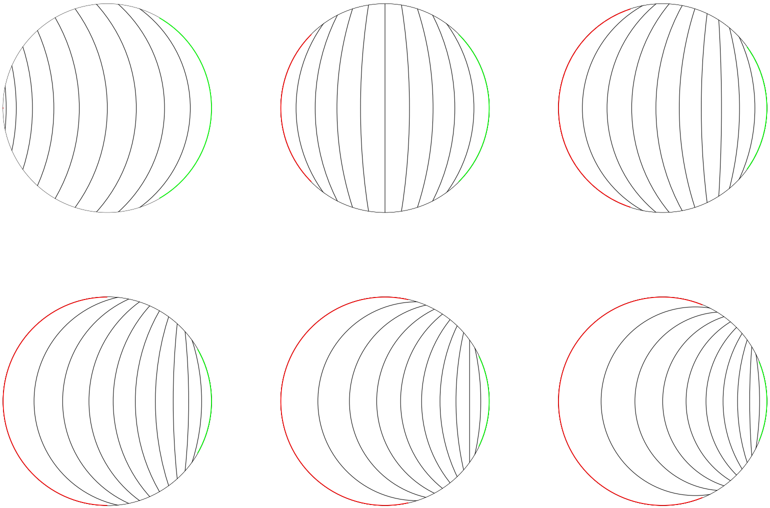
Circular domain

Inverse map

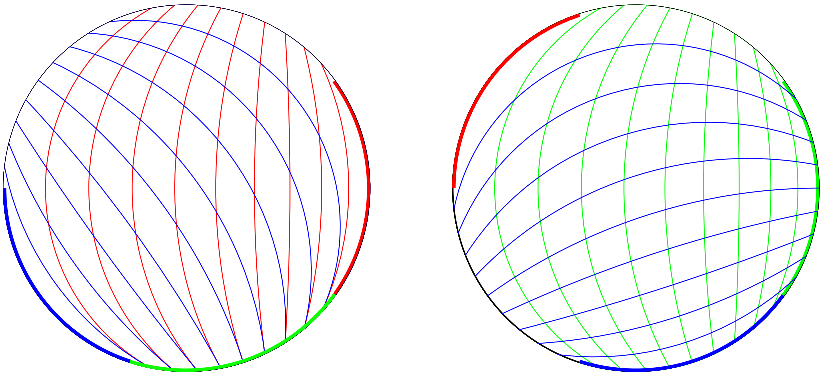


- Unit circle with equal arcs
- Base: $\left[-\frac{\pi}{n}, \frac{\pi}{n}\right]$
- $\varphi = \frac{(2h+1)\pi}{n}$
- $\theta = h\pi \quad \Rightarrow \quad \theta : 0 \to \pi$
- $\psi = \theta - \varphi$
- $\mathbf{O} = \left(\frac{\sin\theta}{\sin\psi}, 0\right)$
- $r = \left|\frac{\sin\varphi}{\sin\psi}\right|$

Examples



Constrained property & corner parameterization

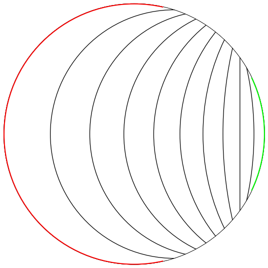


Algorithm

- Line at $\hat{h} = \frac{1}{n-2} \quad \Rightarrow \quad \hat{u} = \cos\frac{\pi}{n-2}$
- Same circle for $h = 0$ and $h = 1$
- Idea: bisection search

if $u > \hat{u}$ **then**
  return `bisection`$(\Delta, 0, \hat{h} - \varepsilon)$
if $u < \hat{u}$ **then**
  return `bisection`$(\Delta, \hat{h} + \varepsilon, 1)$
return $\hat{h}$

$$\Delta(h) = \|\mathbf{p} - \mathbf{O}(h)\| - r(h)$$



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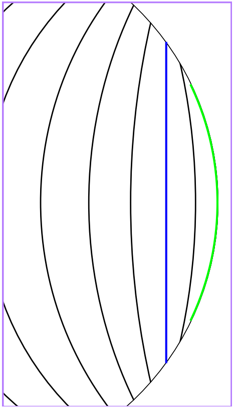
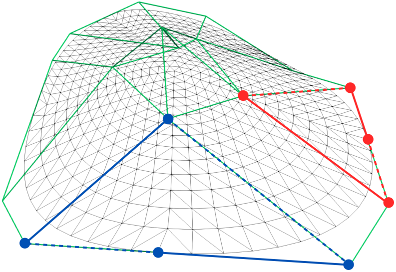


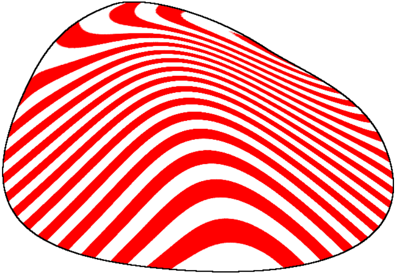
Application

Overlap–GB patch

- Corner-based variation of the GB patch
- Needs a full, constrained parameterization

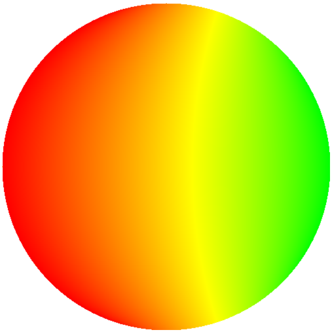
$$S = \sum_{i=1}^{n} \sum_{j=0}^{\lfloor d/2 \rfloor} \sum_{k=0}^{\lfloor d/2 \rfloor} \mathbf{P}_{ijk} B_j^d(h_{i+1}) B_k^d(h_i) + \mathbf{P}_0 B_0,$$





Conclusions

- Circle as multi-sided domain
- Height parameterization
  - Circular arcs
  - Full
  - Constrained
  - Efficient
- Overlap–GB patch
- Suitable for periodic boundaries
- G^2 cap for subdivision surfaces?



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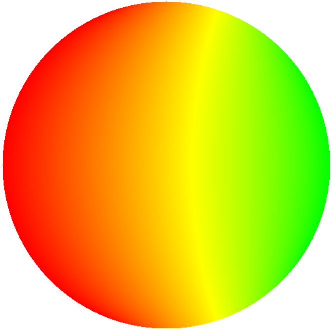

Any questions?