Multi-sided generalizations of the Coons patch

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CSF Workshop on Generalized Barycentric Coordinates

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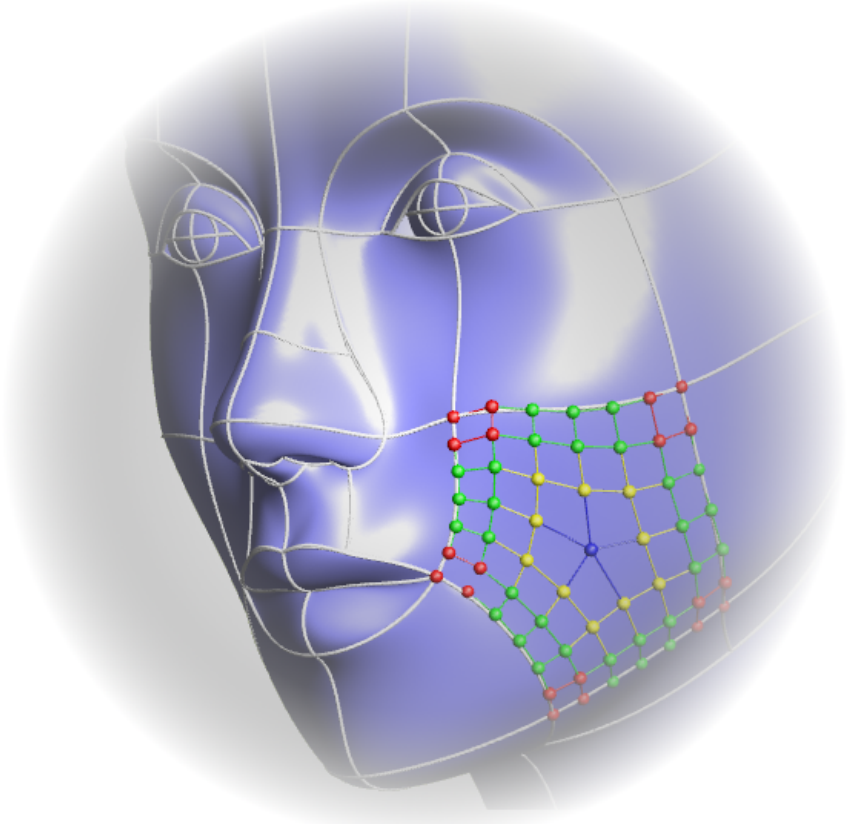
Outline



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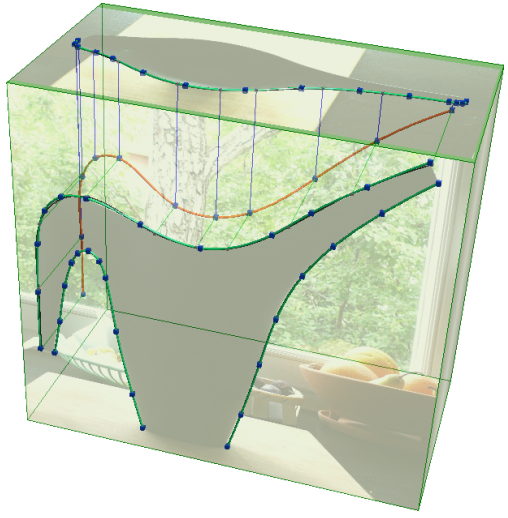
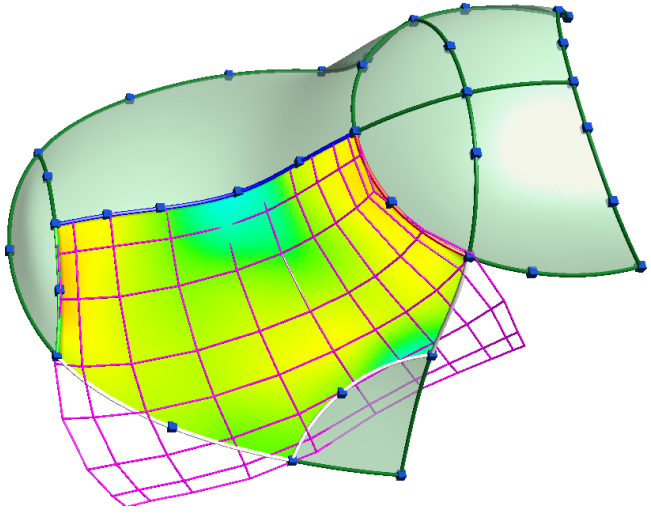
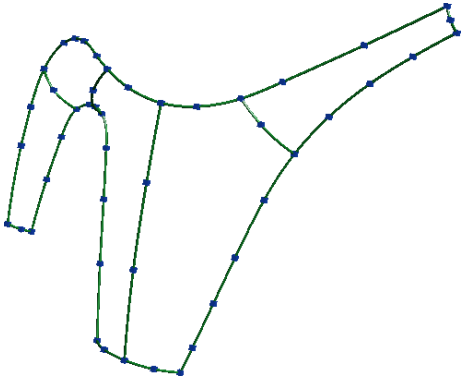
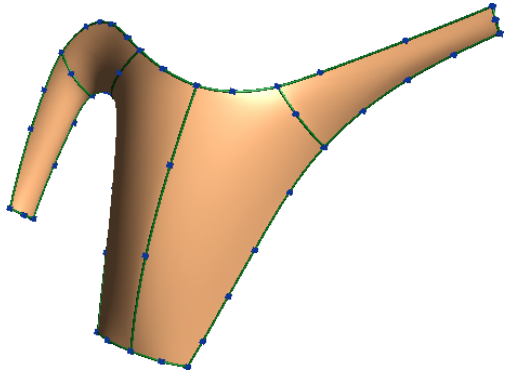
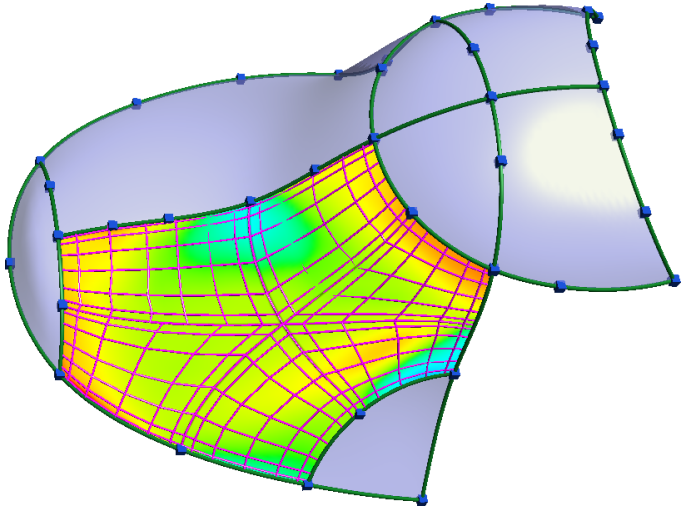
Motivation







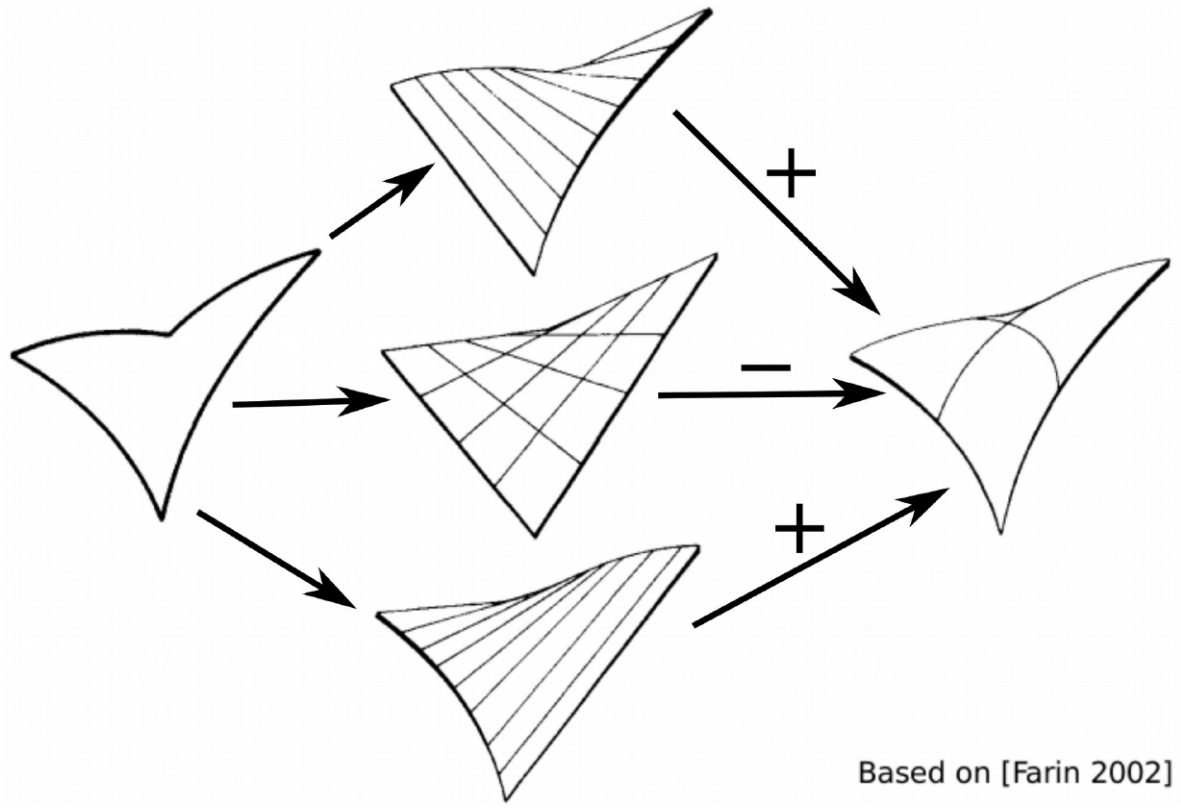




Outline

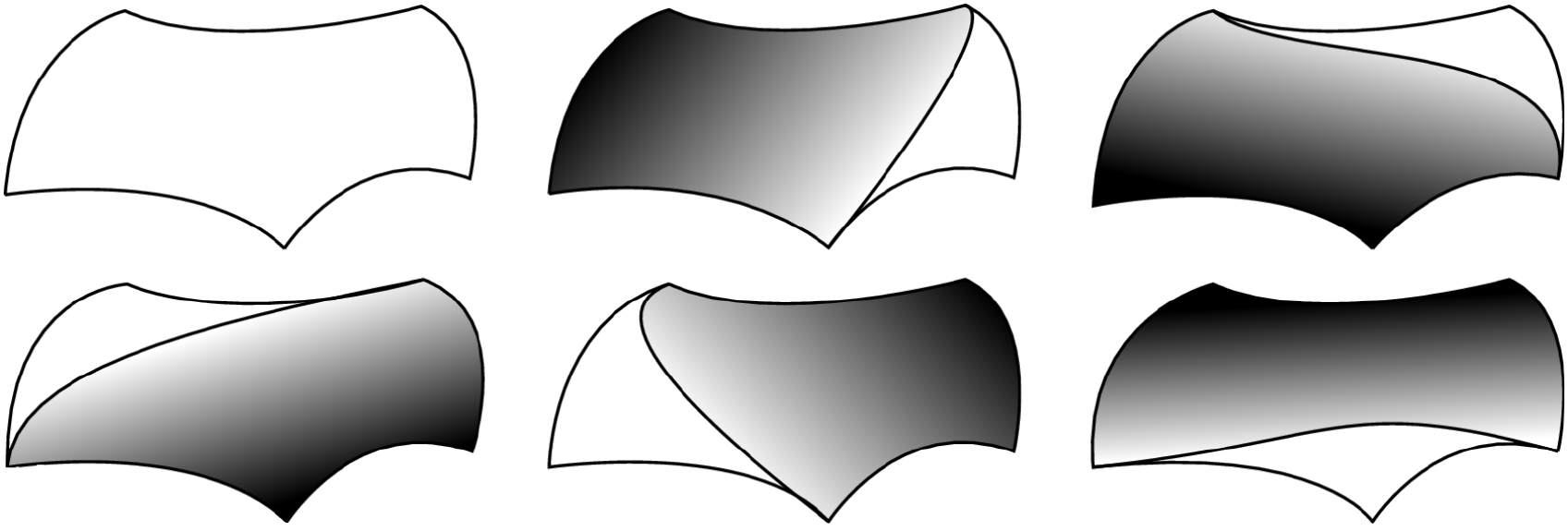
C^0 Coons patch

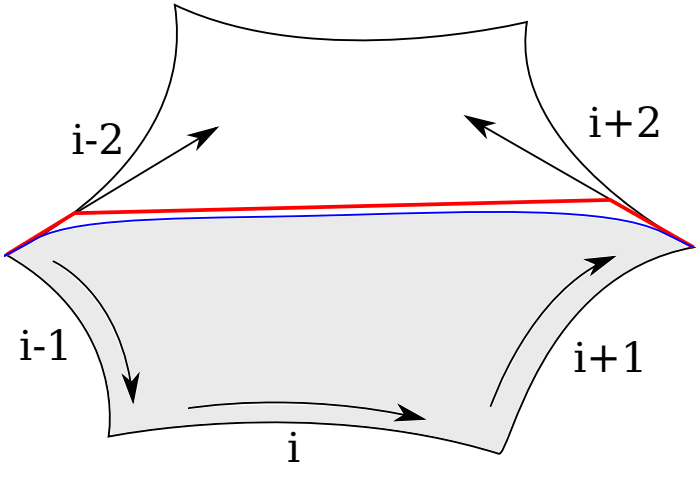
$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 - \mathbf{S}_{12}$$



Outline

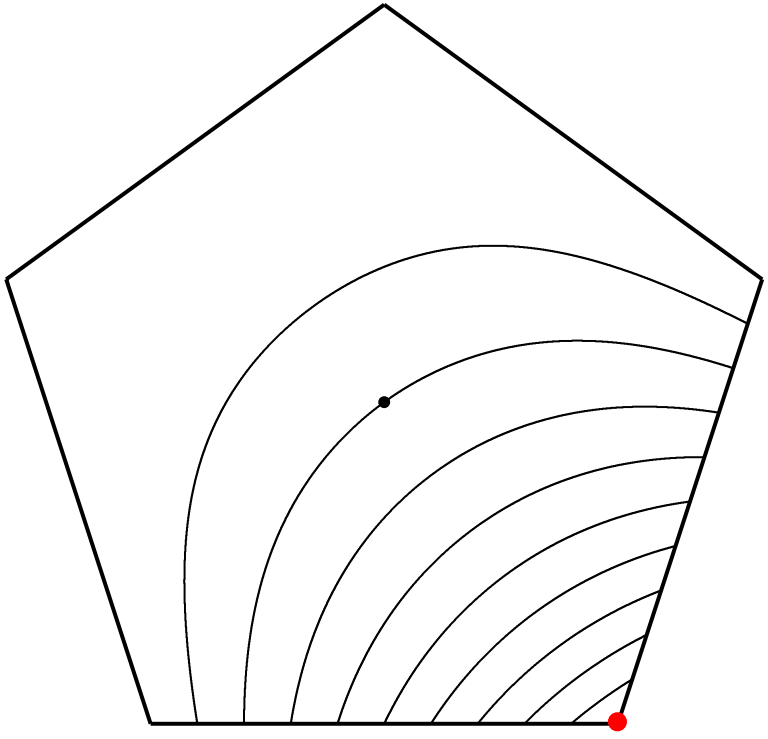
Generalized C^0 Coons patch

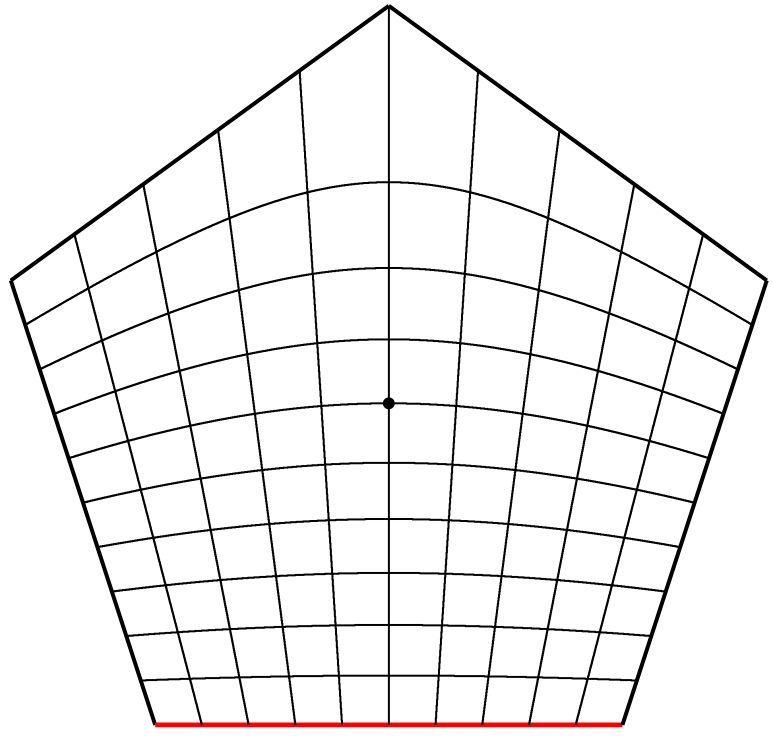




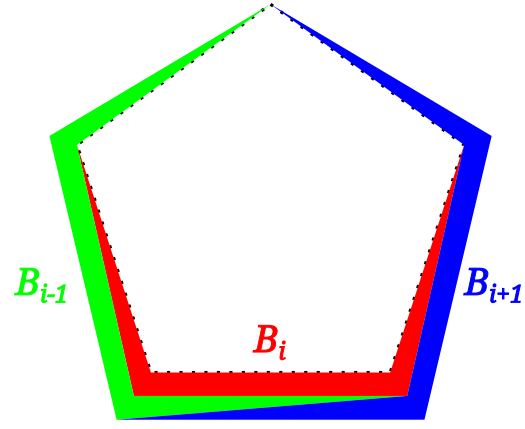
- $\mathbf{S} = \frac{1}{2} \sum_i \mathbf{C}_i^0 B_i^0$
- Domain?
- Local parameterizations?
- Blending function?

Parameterization based on Wachspress coordinates

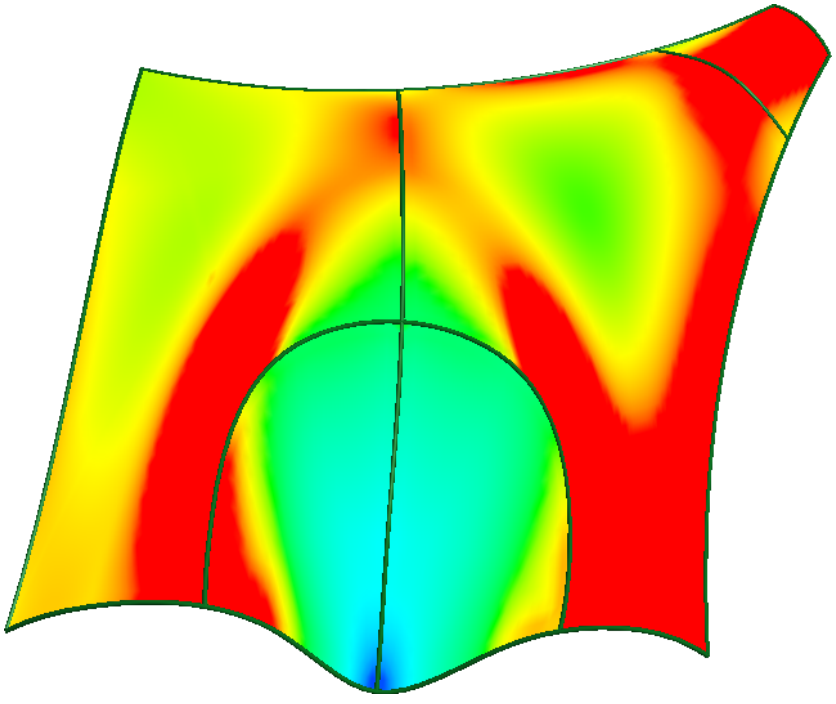


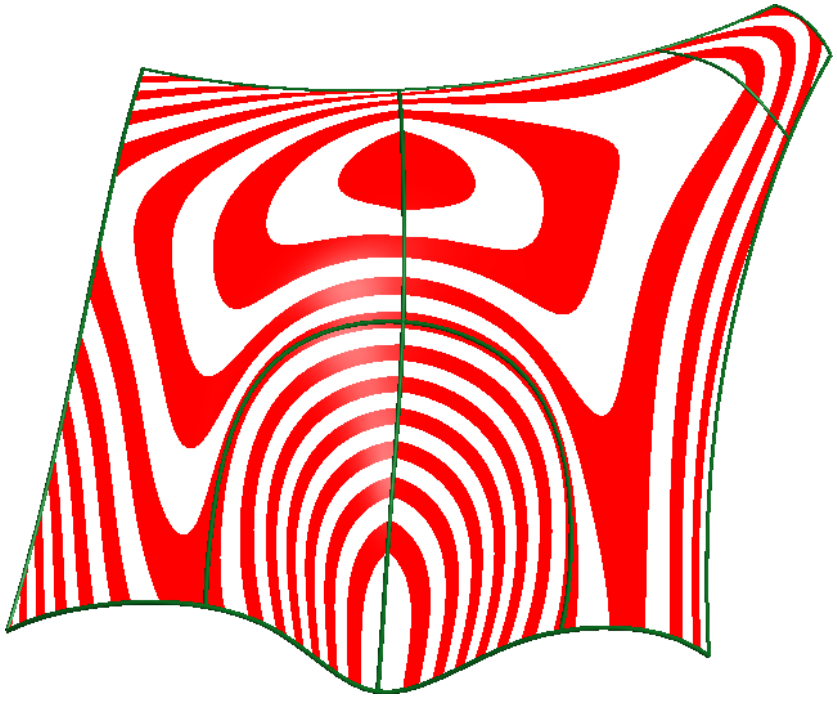


- side parameter $s_i = \lambda_i \,/\, (\lambda_{i-1} + \lambda_i)$
- distance parameter $d_i = 1 - (\lambda_{i-1} + \lambda_i)$
- $B_i^0 = 1 - d_i$



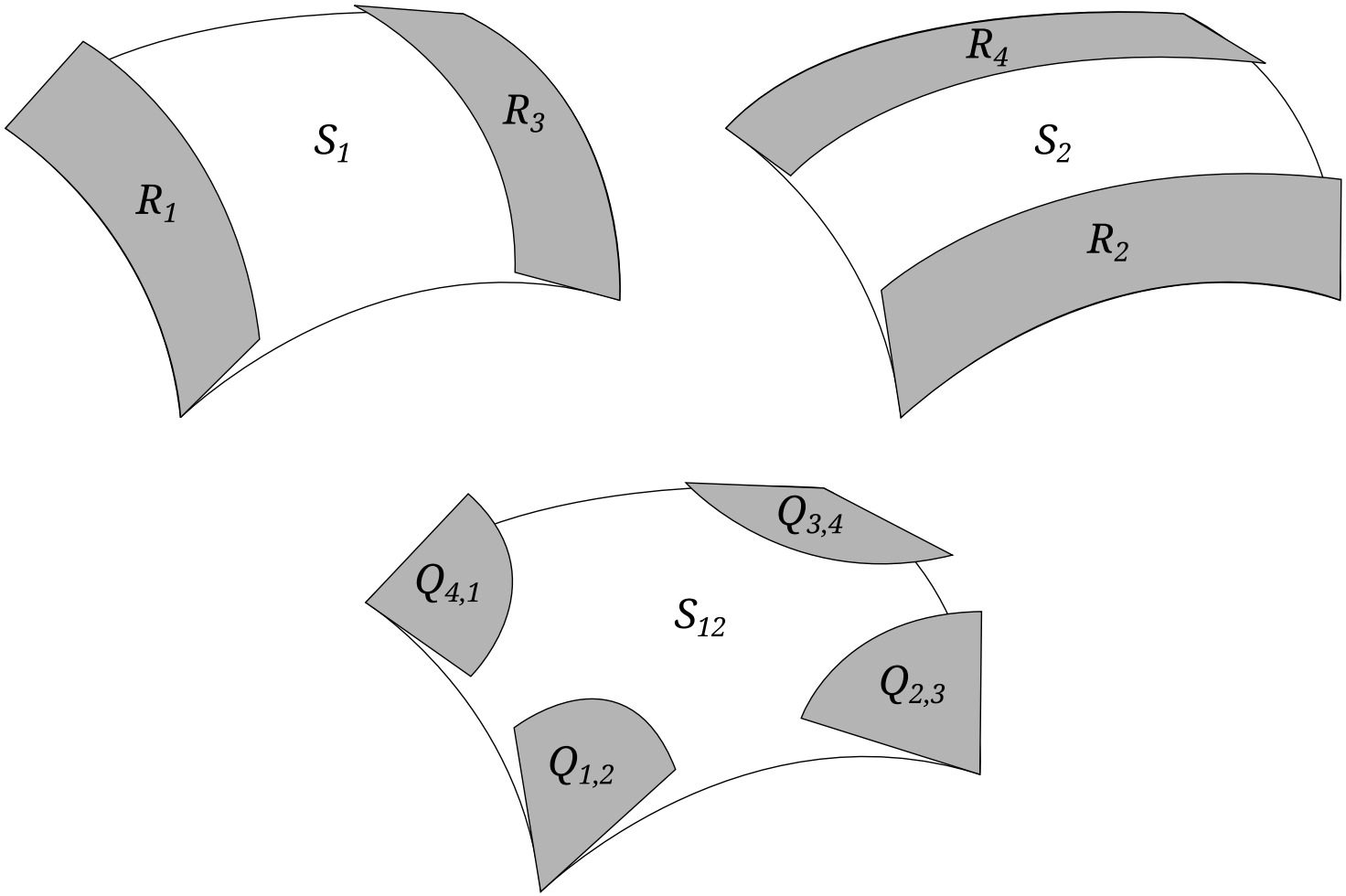
Example (mean curvature & contouring)





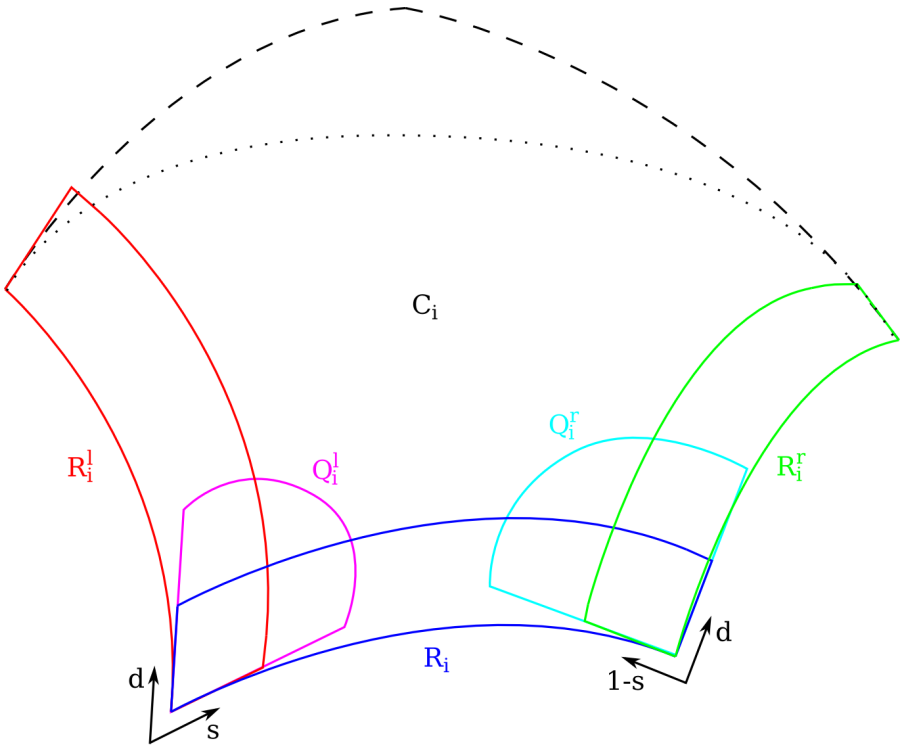
C^1 Coons patch – reinterpreted with blended linear ribbons

$$\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2 - \mathbf{S}_{12}$$

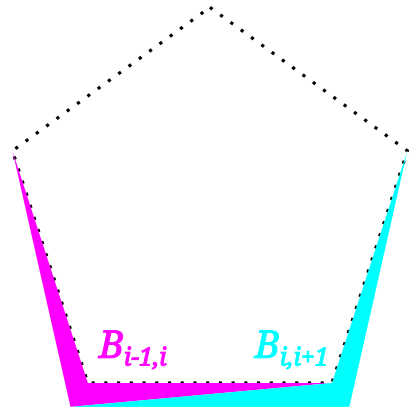


Composite Ribbon patch

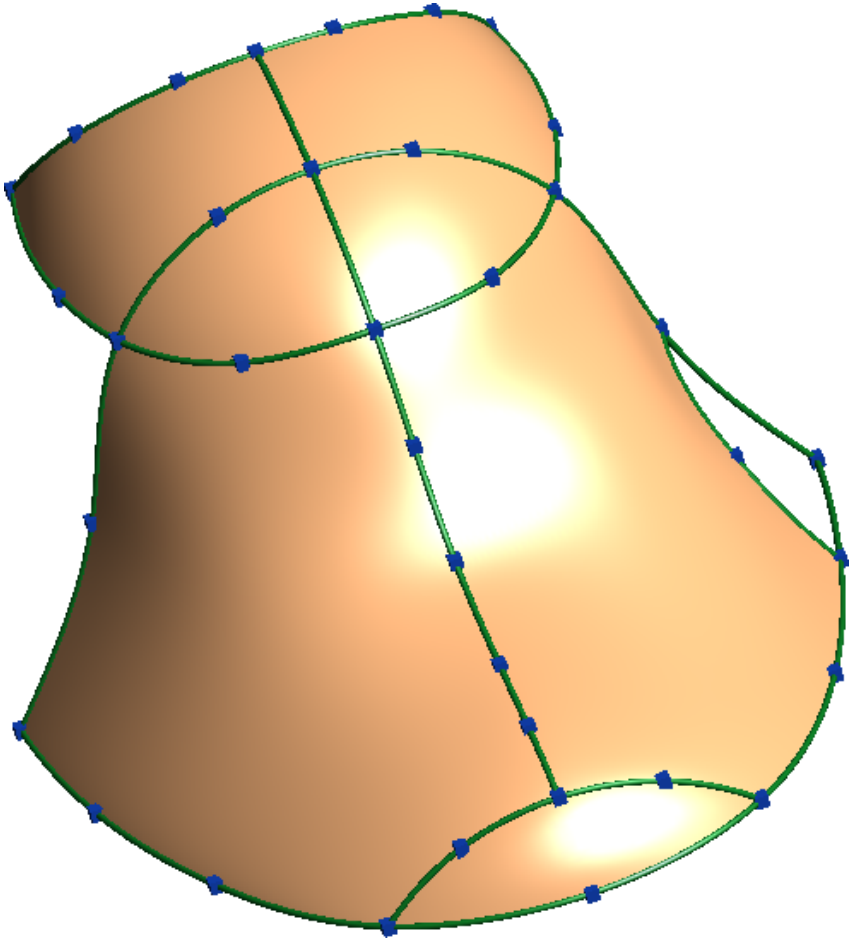
$$\mathbf{S} = \frac{1}{2} \sum_i \mathbf{c}_i B_i$$

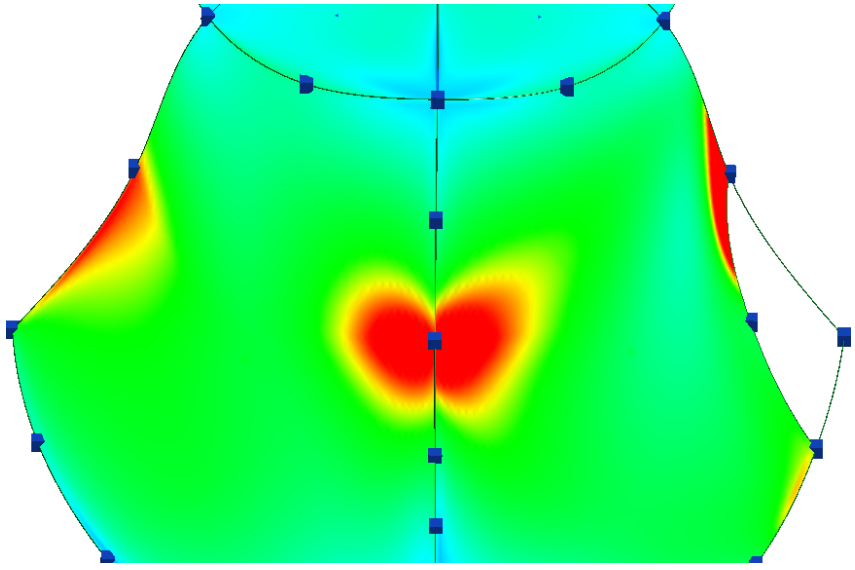


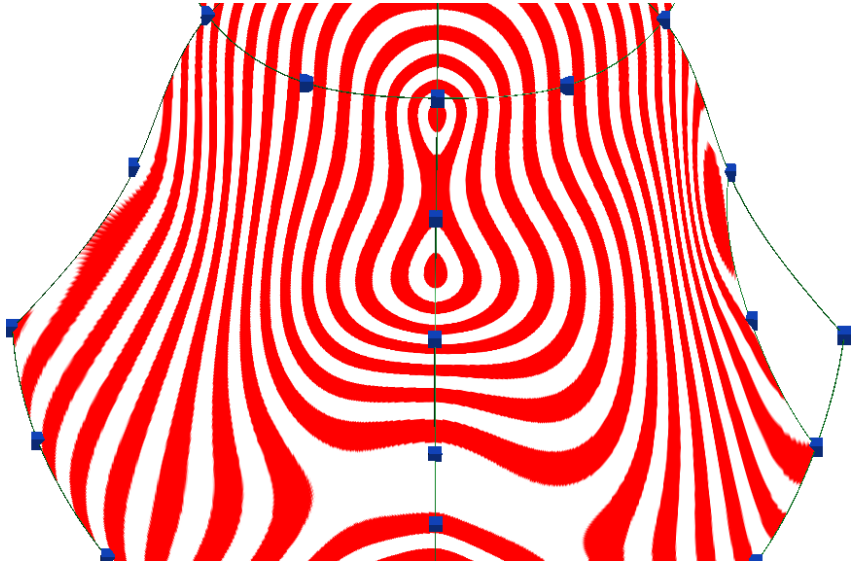
- Blending function? (derivative constraints)
- $B_i = B_{i-1,i} + B_{i,i+1}$
- $B_{i-1,i} = \frac{\prod_{k \notin \{i,i-1\}} d_k^2}{\sum_j \prod_{k \notin \{j,j-1\}} d_k^2}$



Example (mean curvature & isophotes)



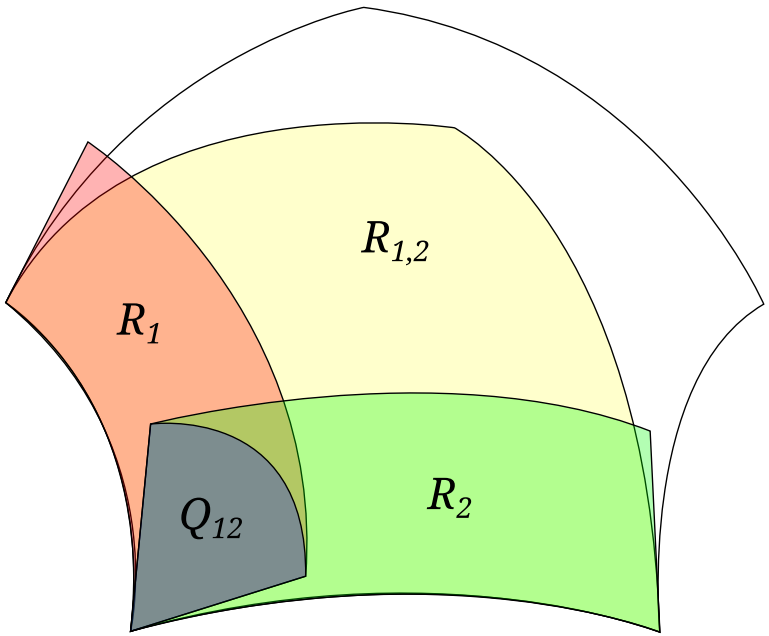


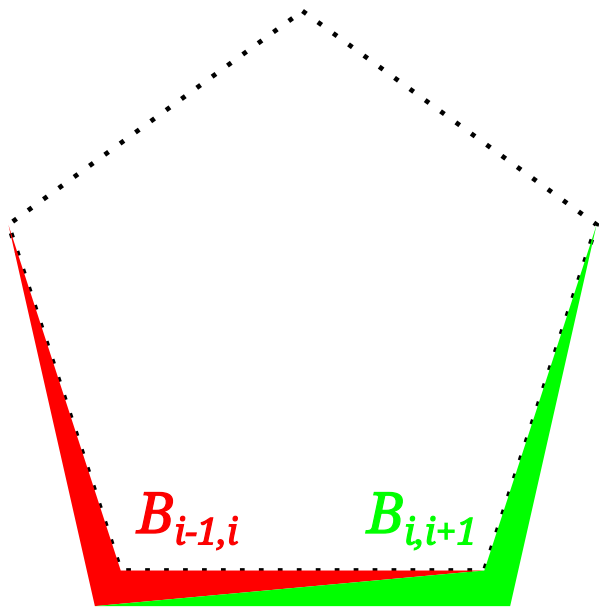


Outline

Charrot–Gregory patch

- Corner interpolant $\approx$ partial Coons patch
- $\mathbf{S} = \sum_i \mathbf{R}_{i-1,i} B_{i-1,i}$
- $\mathbf{R}_{i-1,i} = \mathbf{R}_{i-1} + \mathbf{R}_i - \mathbf{Q}_{i-1,i}$



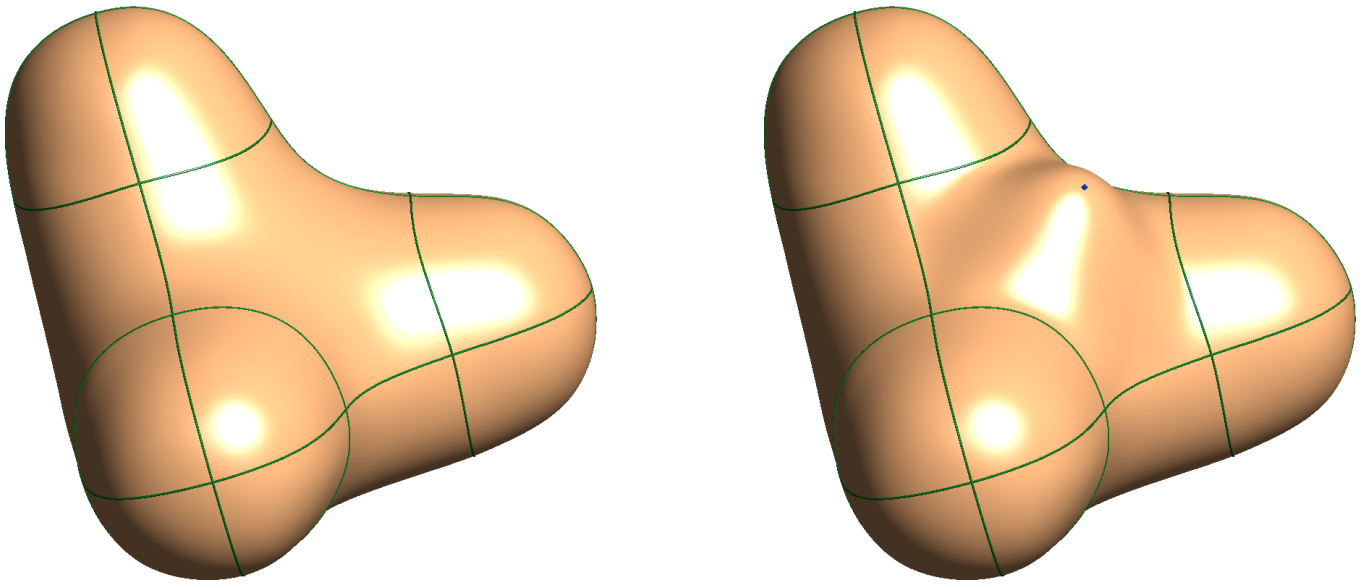


Midpoint patch

- Alternative blending function:

$$B^M_{i-1,i} = \frac{d_{i-1}\alpha_0(s_i)\alpha_0(d_i) + d_i\alpha_1(s_{i-1})\alpha_0(d_{i-1})}{d_{i-1} + d_i}$$

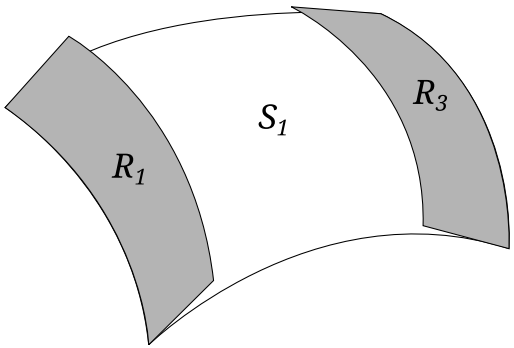
- $\alpha_0(x) = 1 - \alpha_1(x) = 2x^3 - 3x^2 + 1$ (Hermite blends)
- Weight deficient $\rightarrow$ extra DoF

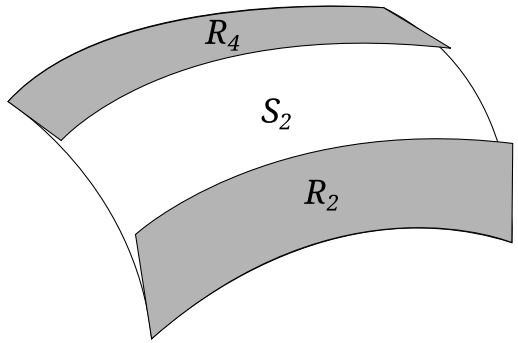


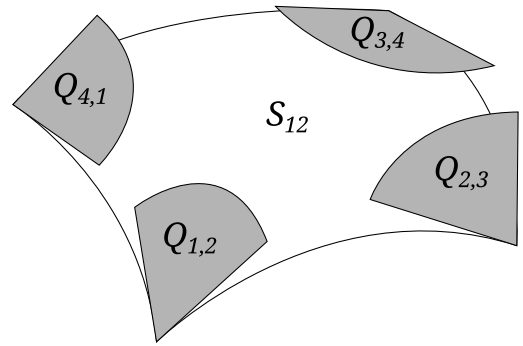
Outline

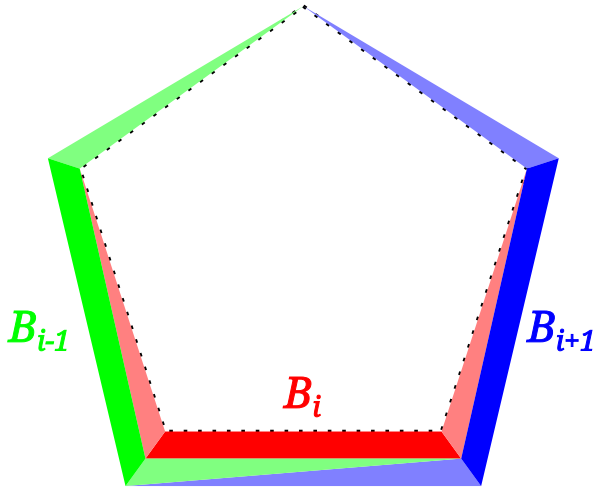
Generalized Coons patch

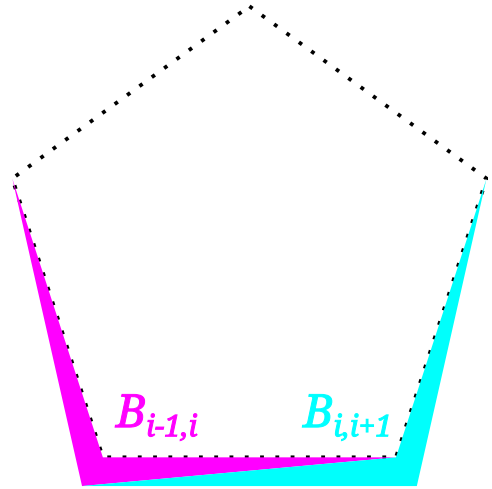
$$\mathbf{S} = \sum_i \mathbf{R}_i B_i - \sum_i \mathbf{Q}_i B_{i-1,i}$$





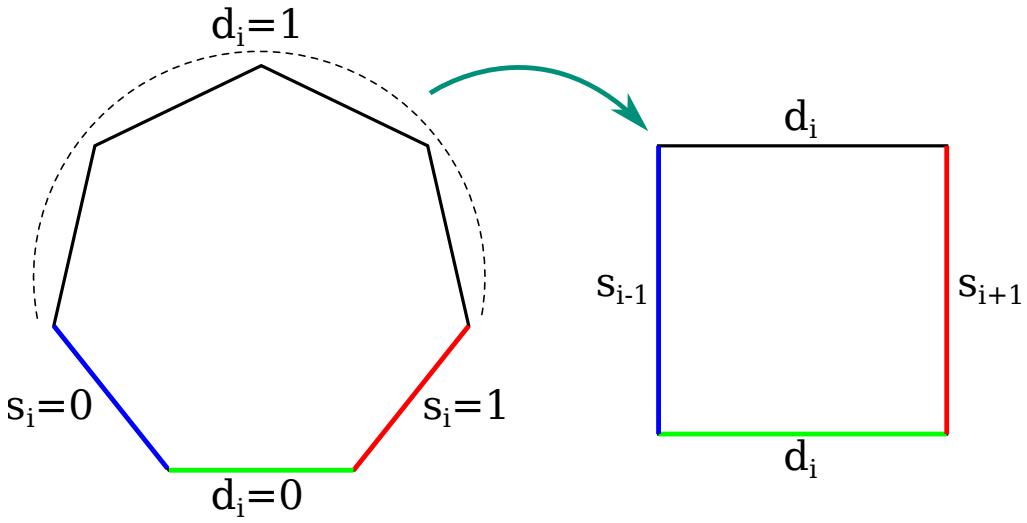




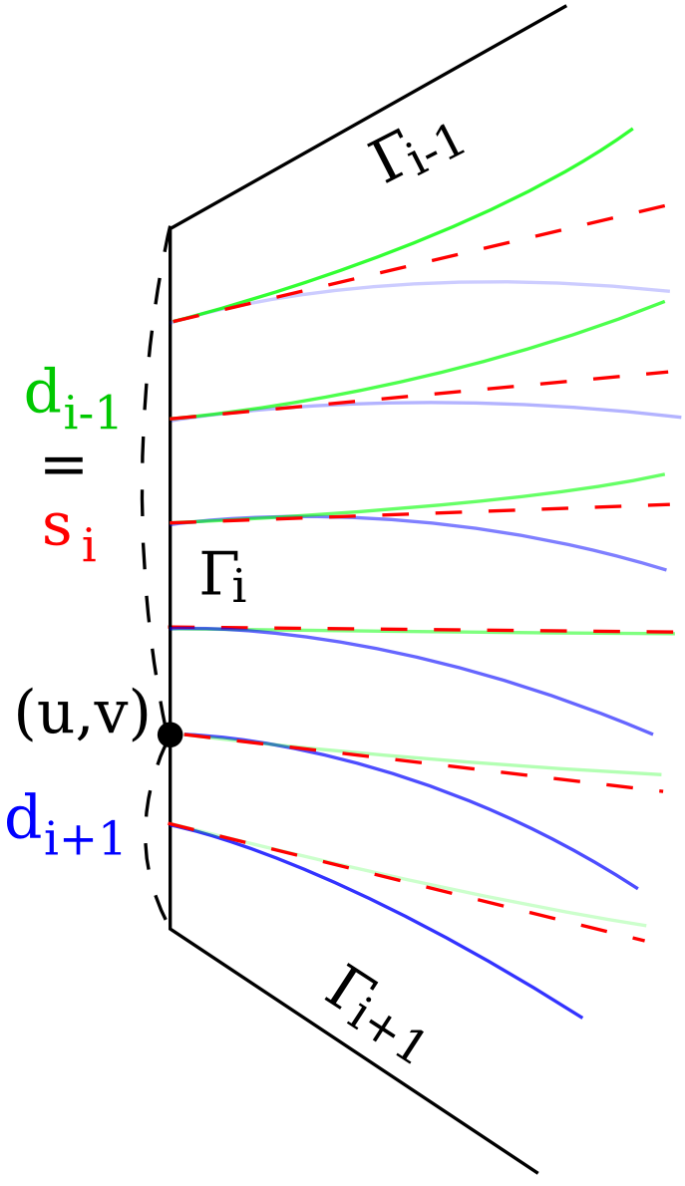


Constrained parameterization

- $d'_{i-1} = s_i = -d'_{i+1}$ on the ith side
- $d(u, v) \approx$ 1D G^1 hole-filling

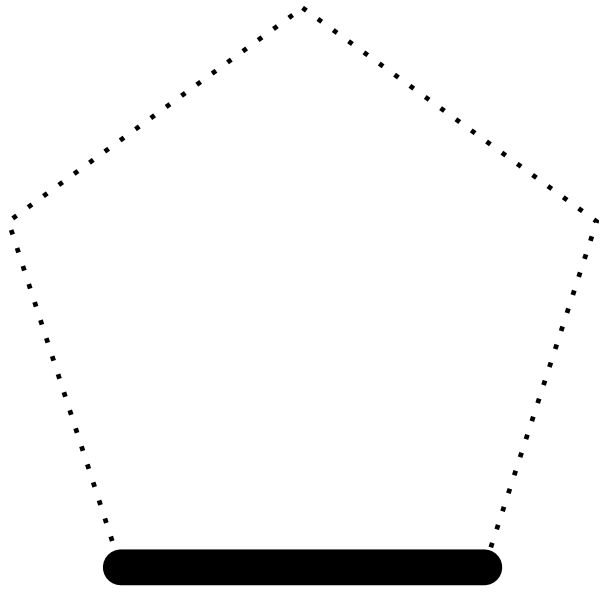


- Use Katō's patch



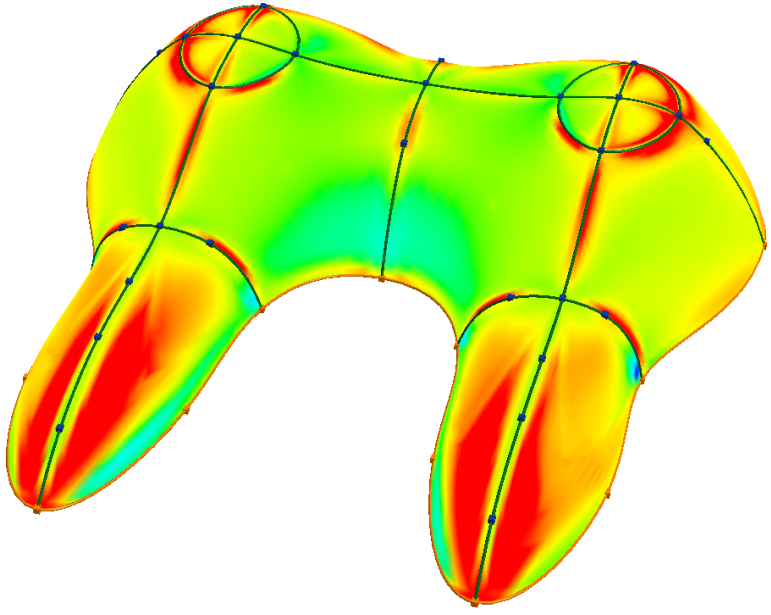
Katō's patch

$$\mathbf{S} = \sum_i \mathbf{R}_i B_i^*$$

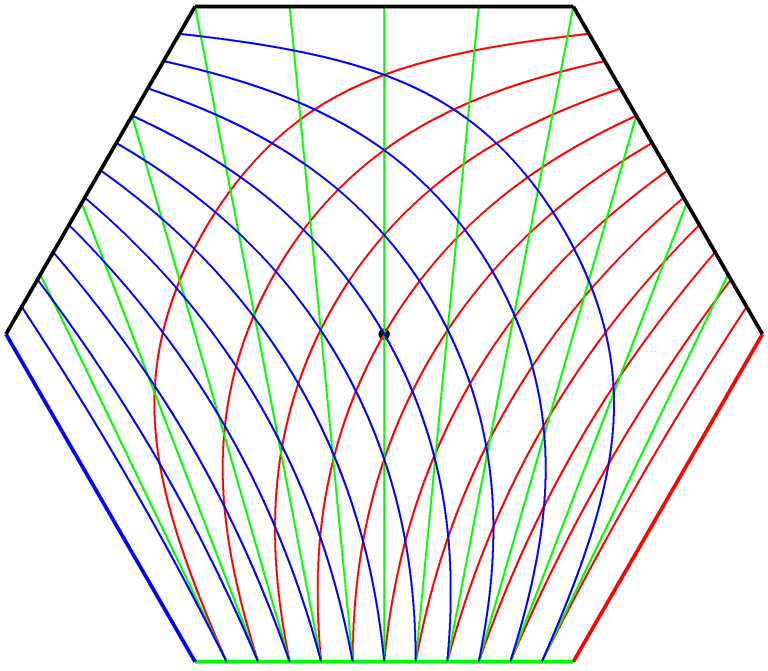


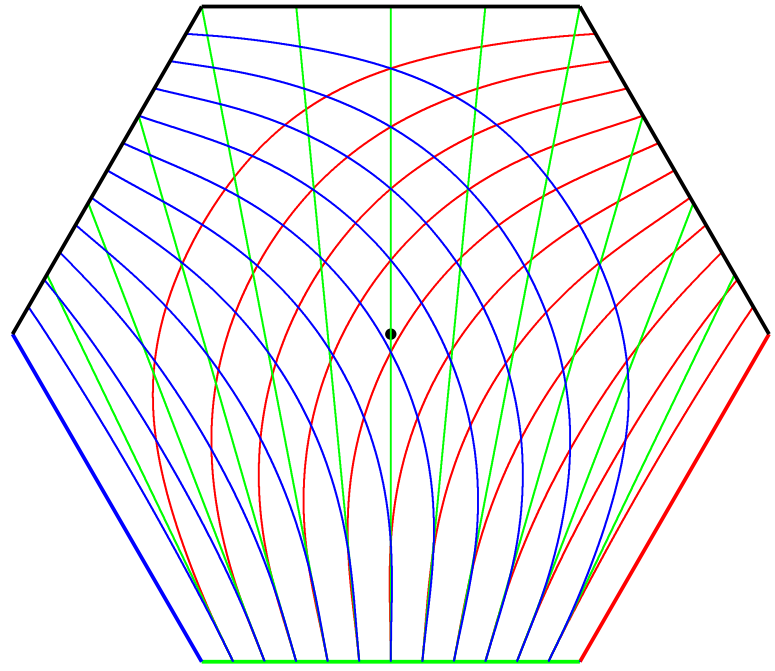
- Singular blending function:

$$B_i^* = \frac{\prod_{k \neq i} d_k^2}{\sum_j \prod_{k \neq j} d_k^2}$$



Constrained parameterization – example



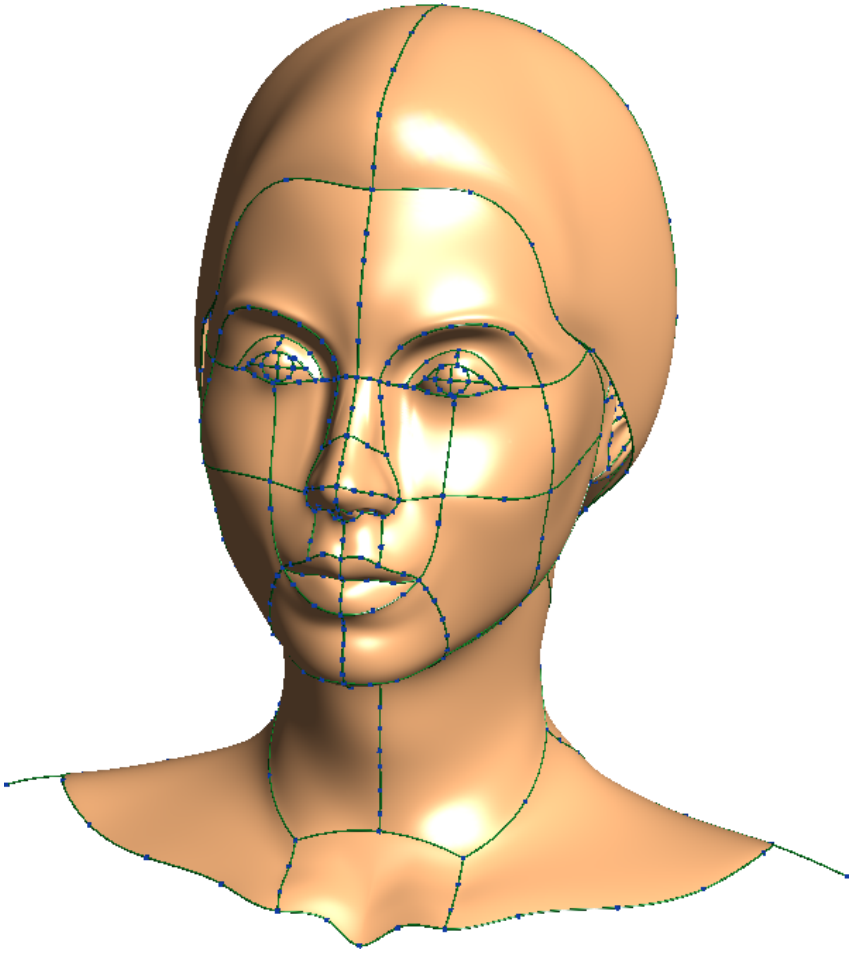


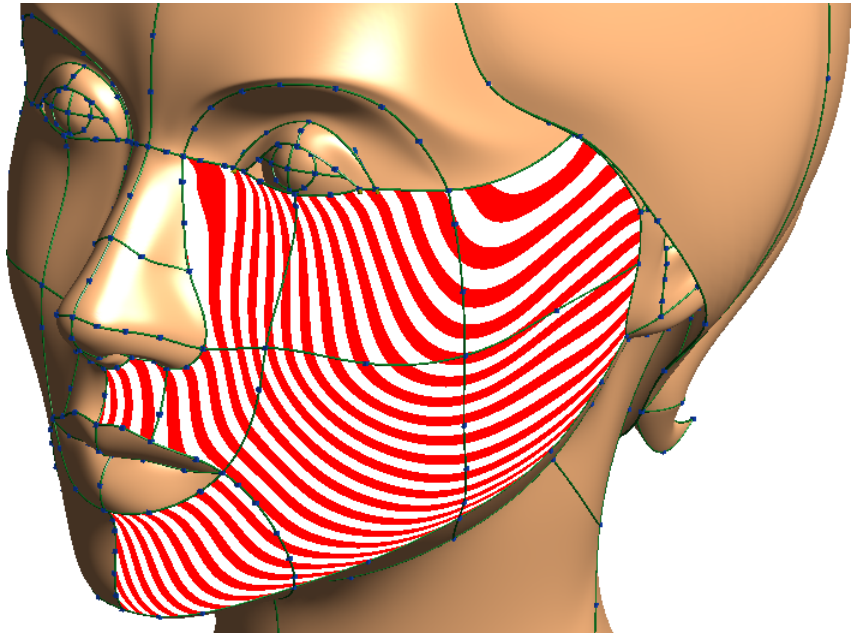
- Midpoint Coons patch:

$$\mathbf{S} = \sum_i \mathbf{R}_i B_i^M - \sum_i \mathbf{Q}_i B_{i-1,i}^M$$

- $B_i^M = B_{i-1,i}^M + B_{i,i+1}^M$

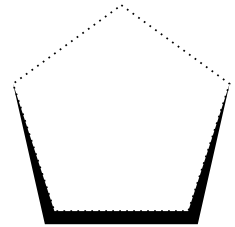
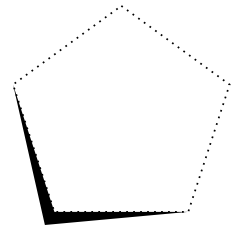
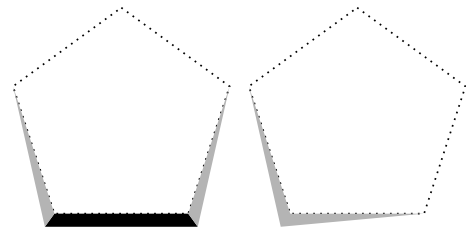
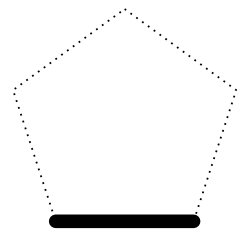
Example (contouring)





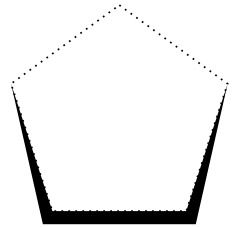
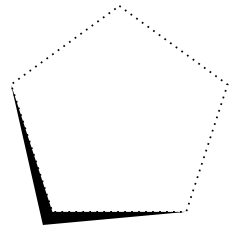
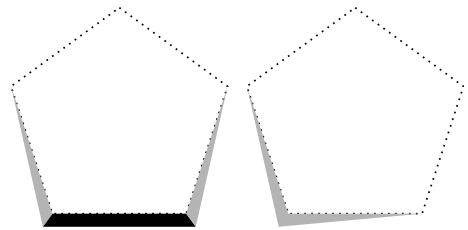
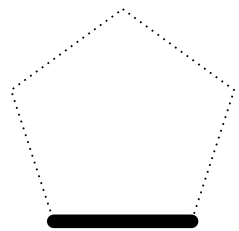
Outline

Summary

Patch type	Ribbon	Parametrization	Blending function
Generalized C^0 Coons	trilat.	full	
Composite ribbon		$d \in [0, 1]$	
Midpoint	bilat.	full	
Charrot–Gregory		simple	
Generalized Coons	unilat.	constrained	
Midpoint Coons		constrained, full	
Katō		simple	

(full: $d = 1$ on the far sides)

Summary

Patch type	Ribbon	Parametrization	Blending function
Generalized C^0 Coons	trilat.	full	
Composite ribbon		$d \in [0,1]$	
Midpoint	bilat.	full	
Charrot–Gregory		simple	
Generalized Coons	unilat.	constrained	
Midpoint Coons		constrained, full	
Katō		simple	

(full: $d = 1$ on the far sides)

Related papers

1. Generalized Coons & Composite Ribbon patches:

   P. Salvi, T. Várady, A. Rockwood, *Ribbon-based transfinite surfaces*. **Computer Aided Geometric Design**, Vol. 31(9), pp. 613–630, 2014.

2. Midpoint patch:

   P. Salvi, T. Várady, *Multi-sided surfaces with fullness control*. Proceedings of the Eighth Hungarian Conference on Computer Graphics and Geometry, pp. 61–69, 2016.

3. Midpoint Coons patch:

   P. Salvi, I. Kovács, T. Várady, *Computationally efficient transfinite patches with fullness control*. Proceedings of the Workshop on the Advances of Information Technology, pp. 96–100, 2017.

4. Generalized C^0 Coons patch:

   P. Salvi, *A multi-sided generalization of the C^0 Coons patch*. Proceedings of the Workshop on the Advances of Information Technology, pp. 110–111, 2020.