Using Bézier extraction matrices

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Given a B-spline curve of degree d with L segments and n control points in the form

$$\mathcal{C}(u) = \sum_{k=0}^{n-1} \mathbf{P}_k N_k^{d,U}(u),$$

where U is the clamped knot vector with $d+1$ identical values at both ends, and $N_k^{d,U}$ are the B-spline basis functions, the i-th Bézier component ($i = 0..L-1$) can be extracted as

$$\hat{\mathcal{C}}_i(u) = \sum_{j=0}^{d} \mathbf{Q}_j^i B_j^d(u),$$

where B_j^d are the Bernstein polynomials and

$$\mathbf{Q}_j^i = \sum_{k=0}^{d} \mathbf{P}_{k+s_i-d} C_{k,j}^i \quad \equiv \quad [\mathbf{Q}^i] = \mathbf{C}^{i^\mathsf{T}} \cdot [\mathbf{P}],$$

because of the extraction relation

$$N_k^{d,U}(u) = \sum_{j=0}^{d} C_{k+d-s_i,j}^i B_j^d(u),$$

$\mathbf{C}^i$ being the i-th extraction matrix of the degree-d knot vector U, and s_i denoting the span index of the i-th segment.

Conversely, a B-spline control point $\mathbf{P}_k$ can be restored from a suitable Bézier segment (i.e., one that is in the same span), using the formula

$$\mathbf{P}_k = \sum_{j=0}^{d} \mathbf{Q}_j^i \left(\mathbf{C}^{i^{-1}}\right)_{j,k+d-s_i} \quad \equiv \quad [\mathbf{P}] = \left(\mathbf{C}^{i^{-1}}\right)^\mathsf{T} \cdot [\mathbf{Q}^i].$$