Using Bézier extraction matrices

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Given a B-spline curve of degree $d$ with $L$ segments and $n$ control points in the form

$$\mathcal{C}(u) = \sum_{k=0}^{n-1} P_k N_k^{d,U}(u),$$

where $U$ is the clamped knot vector with $d+1$ identical values at both ends, and $N_k^{d,U}$ are the B-spline basis functions, the $i$-th Bézier component ($i = 0..L - 1$) can be extracted as

$$\hat{\mathcal{C}}_i(u) = \sum_{j=0}^{d} Q_{i,j}^d B_j^d(u),$$

where $B_j^d$ are the Bernstein polynomials and

$$Q_{i,j}^d = \sum_{k=0}^{d} P_{k+s_i-d} C_{k,j}^i \equiv [Q_i^d] = C_i^T \cdot [P],$$

because of the extraction relation

$$N_k^{d,U}(u) = \sum_{j=0}^{d} C_{k+d-s_i,j}^i B_j^d(u),$$

$C_i^i$ being the $i$-th extraction matrix of the degree-$d$ knot vector $U$, and $s_i$ denoting the span index of the $i$-th segment.

Conversely, a B-spline control point $P_k$ can be restored from a suitable Bézier segment (i.e., one that is in the same span), using the formula

$$P_k = \sum_{j=0}^{d} Q_{j}^d (C_i^{-1})_{j,k+d-s_i} \equiv [P] = (C_i^{-1})^T \cdot [Q_i^d].$$