

Fudging Fudge Dice

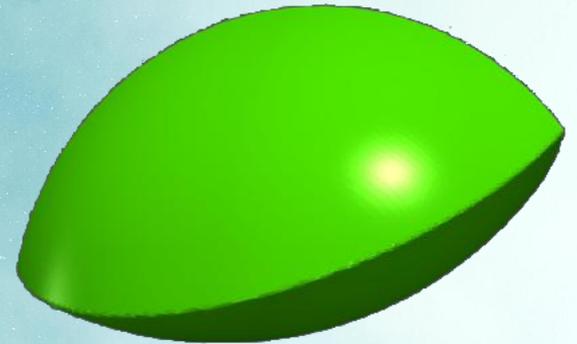
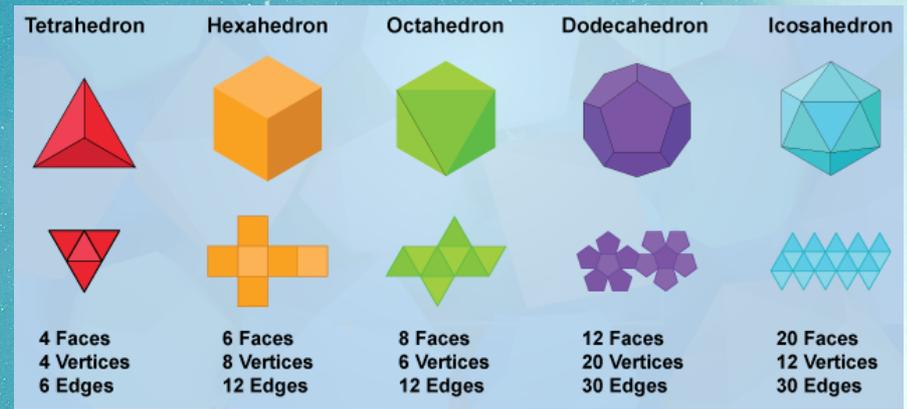
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WAIT 2026

Budapest, 2026.02.10.







d6						
dF(a)						
dF(b)						
dF(c)						

<i>x</i>	-4	-3	-2	-1	0	1	2	3	4
Ratio	1	4	10	16	19	16	10	4	1
≈ (%)	1.23	4.94	12.35	19.75	23.46	19.75	12.35	4.94	1.23
Ratio	11	45	120	210	252	210	120	45	11
≈ (%)	1.07	4.39	11.72	20.51	24.61	20.51	11.72	4.39	1.07

clamp(Heads - 5, -4, 4)

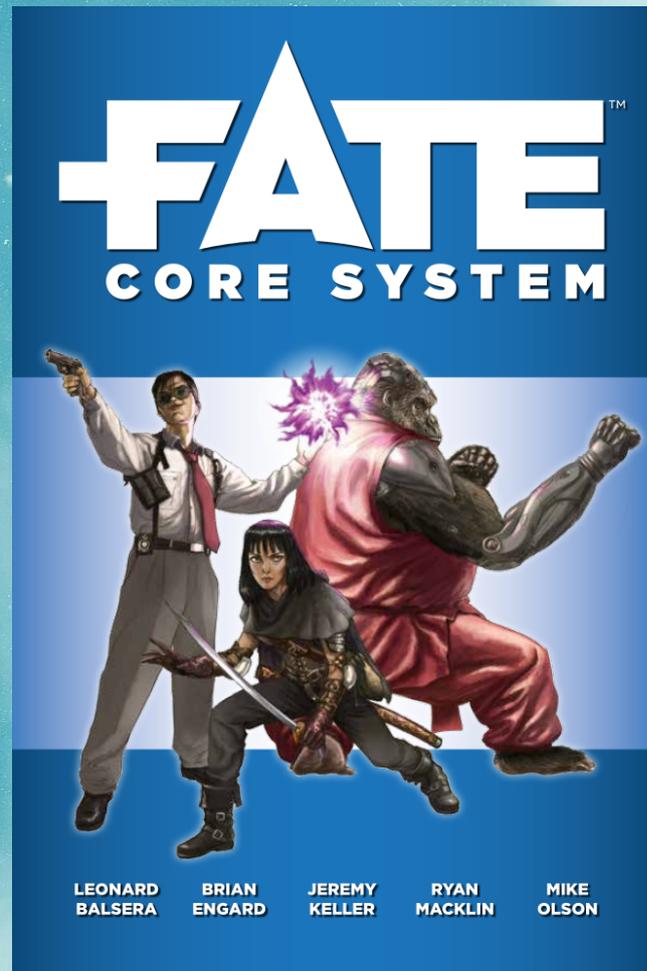


Expected χ^2 : 0.00339 *N*

DoF: 8, α : 0.05

Critical value: 15.507

For *N*=1000, not rejected
in >75% of the cases





d6						
dF(a)						
dF(b)						
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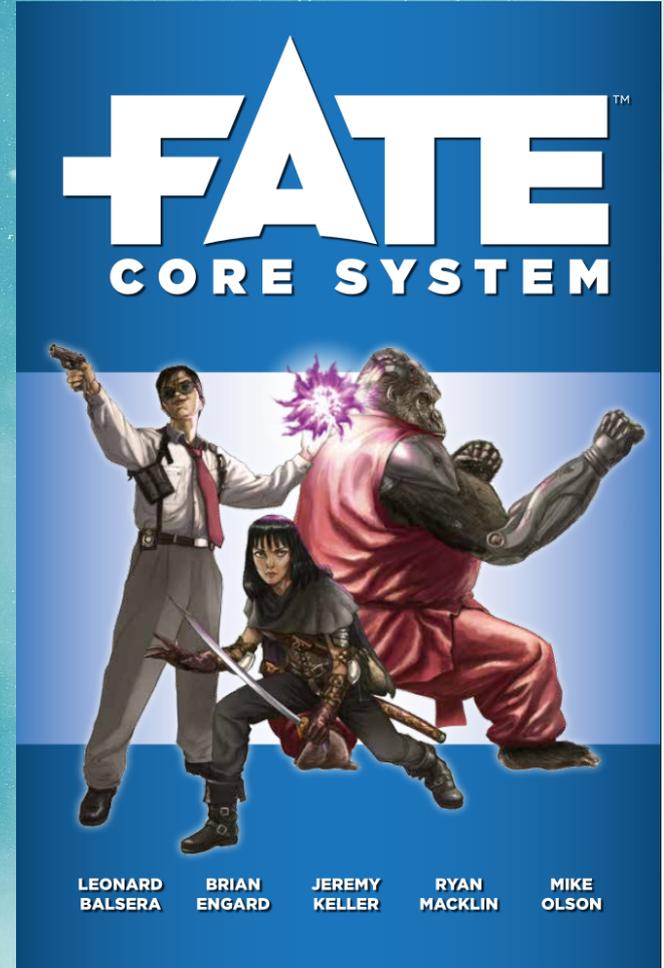
x	-4	-3	-2	-1	0	1	2	3	4
Ratio	1	4	10	20	25	20	10	4	1
≈ (%)	1.23	4.94	11.72	24.61	28.75	24.61	11.72	4.94	1.23
Ratio	11	45	117	246	288	246	117	45	11
≈ (%)	1.07	4.39	11.72	24.61	28.75	24.61	11.72	4.39	1.07

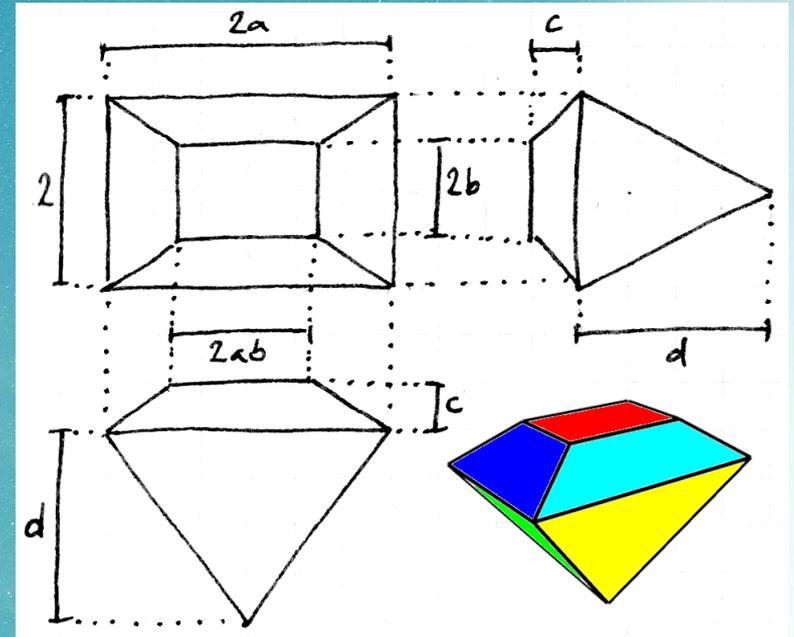


$\text{clamp}(\text{Heads} - 5, -4, 4)$

Expected χ^2 : 0.00339 N
 DoF: 8, α : 0.05
 Critical value: 15.507

For $N=1000$, not rejected
 in >75% of the cases



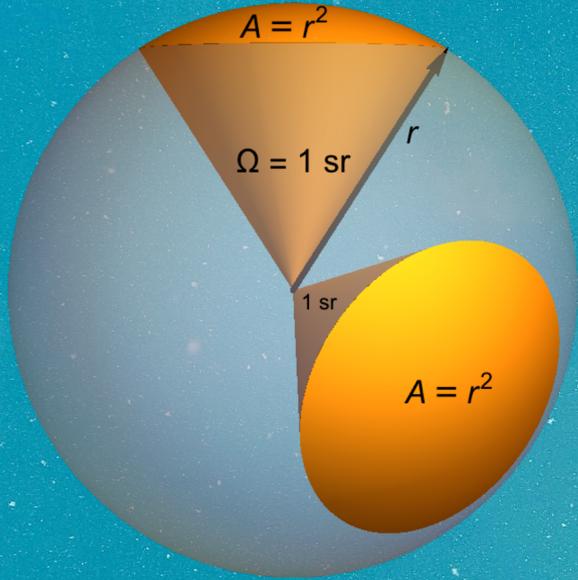


Parameter	Minimum	Maximum	Description
a	1	2	Aspect ratio
b	0.3	0.9	Top shrinkage
c	0.1	1	Top height
d	0.5	4	Bottom depth

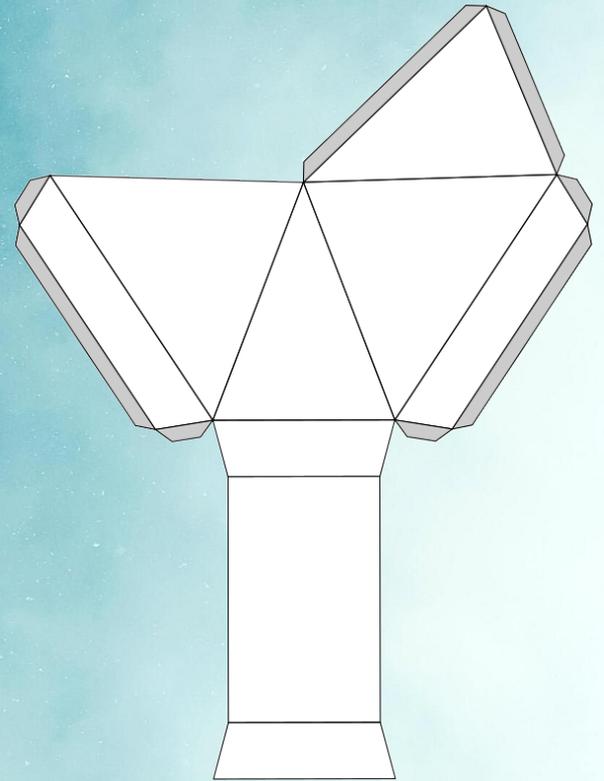
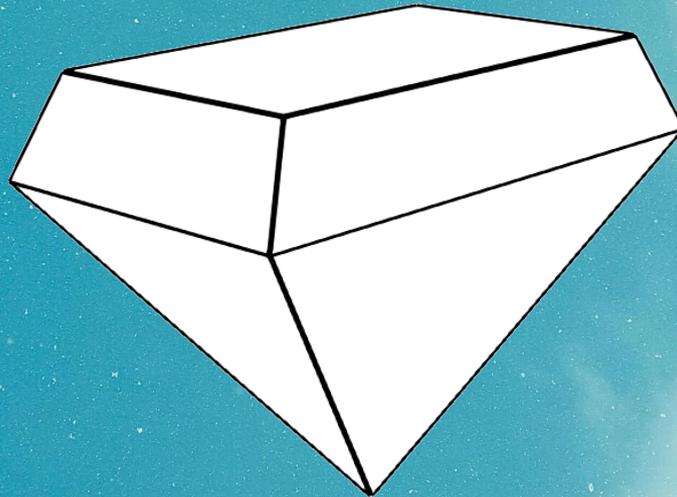
$$\text{COM}_y = \frac{c^2(3b^2 + 2b + 1) - d^2}{4(c(b^2 + b + 1) + d)}$$

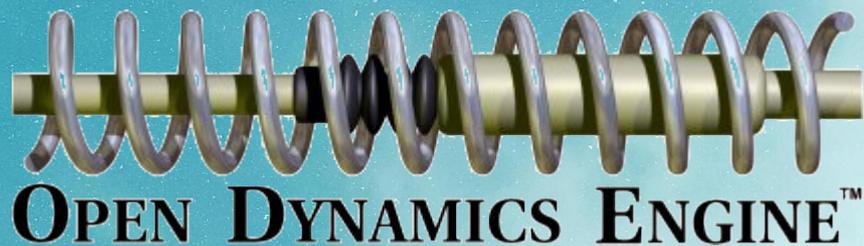
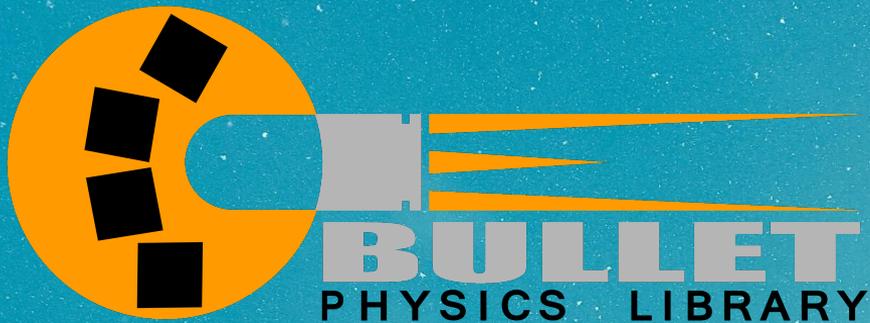
Solid angle ratio ~ landing ratio?

Optimize
$$\sum_{k=-4}^4 \frac{(q_k - p_k)^2}{p_k}$$



$a = 1.61$
 $b = 0.83$
 $c = 0.56$
 $d = 2.04$





- Falling from ~5 units height
 - Random orientation
 - Random angular velocity
 - Restitution: 0.3 (bouncy)
 - Friction: 0.8 (moderate)
- 1000 simulations / evaluation

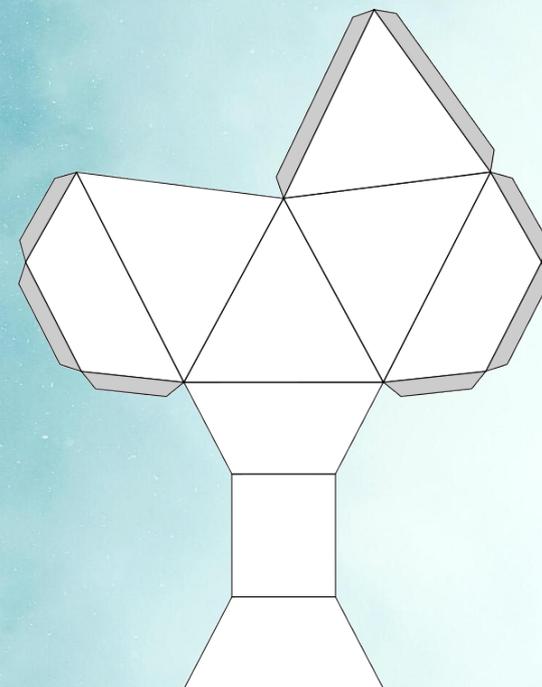
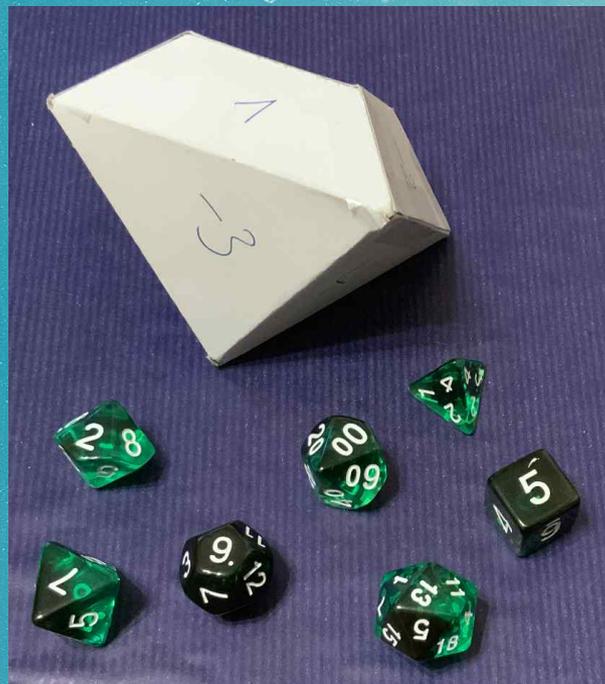
$a = 1.18$

$b = 0.52$

$c = 0.72$

$d = 1.41$

After 30000 tests:
slightly favors ± 1 .



Real-Life Experiment (2×405 rolls)



		Paper die		3D-printed die	
Value	Target	Count	Error	Count	Error
-4	5	0	-100%	7	+40%
-3	20	20	0%	33	+65%
-2	50	48	-4%	49	-2%
-1	80	80	0%	76	-5%
0	95	89	-6%	88	-7%
1	80	83	+4%	80	0%
2	50	66	+32%	49	-2%
3	20	17	-15%	21	+5%
4	5	2	-60%	2	-60%

Die type	χ^2	p	TVD	JSD
Paper	12.97 ✓	0.11 ✓	0.047 ✓	0.0068 ✓
3D-printed	11.86 ✓	0.16 ✓	0.040 ✓	0.0033 ✓

3 Metrics:

- χ^2 value:

$$N \cdot \sum_{k=-4}^4 \frac{(p_k - \hat{q}_k)^2}{p_k}$$

- Total Variation Distance (TVD):

$$\frac{1}{2} \sum_{k=-4}^4 |p_k - \hat{q}_k|$$

- Jensen-Shannon Divergence (JSD):

$$\frac{1}{2} D_{KL}(P||M) + \frac{1}{2} D_{KL}(\hat{Q}||M)$$

where

$$D_{KL}(P||M) = \sum_{i=-4}^4 p_i \ln \frac{p_i}{m_i}$$

is the Kullback-Leibler divergence, and

$$M = (P + \hat{Q})/2$$

Acknowledgements

- Gemini 3
 - Solid angle based experiments
 - Statistical analysis
 - General discussions
- Anthropic Claude Sonnet 4.5
 - Bullet Physics simulation
- Grok 4
 - Open Dynamics Engine simulation
- ChatGPT 5.2, Microsoft Copilot & DeepSeek
 - Statistical analysis
 - General discussions



Gemini



Claude



Grok

