Enhancement of a multi-sided Bézier surface representation

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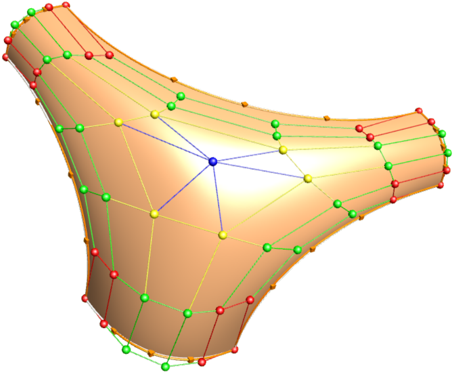
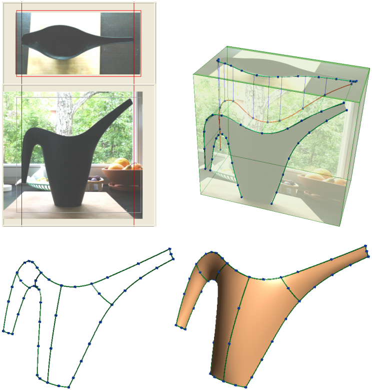
5. Conclusion and future work



Applications of multi-sided patches

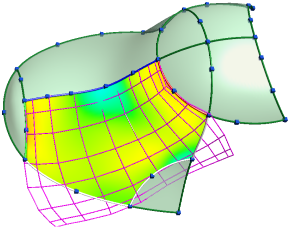
- Curve network based design
  - Feature curves
  - Automatic surface generation
- Hole filling
  - E.g. vertex blends
  - Cross-derivative constraints
- 3D point cloud approximation
  - Given boundary loops
  - Smoothly connected patches

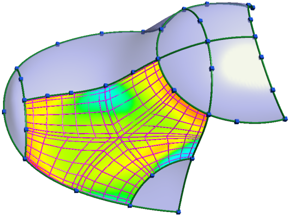
Representation?



Conventional representations

- Trimmed/split tensor product surfaces
  - Detailed control in the interior
  - CAD-compatible
  - But: continuity problems
- Recursive subdivision
  - Arbitrary topology
  - Easy to design with
  - But: hard to interpolate boundary cross-derivatives
- Transfinite patches
  - Interpolates any number of sides
  - Depends only on the boundary
  - But: little interior control



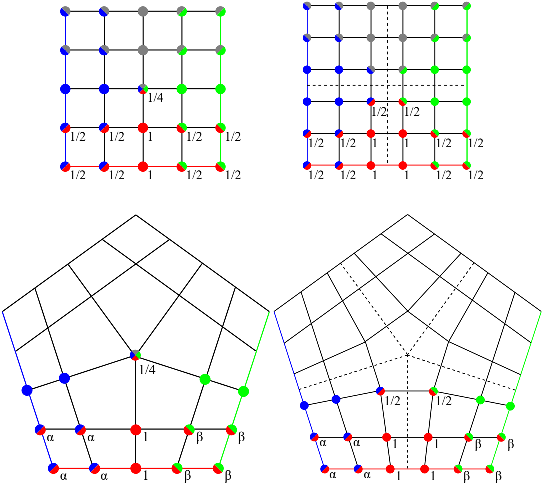


Multi-sided surfaces with control networks

- Loop and DeRose (1989)
  - S-patches – beautiful theory, difficult to use
- Warren (1992)
  - Based on Bézier triangles, max. 6 sides
- Zheng and Ball (1997)
  - High-degree expressions, max. 6 sides
- Krasauskas (2002)
  - Toric patches – lattice-based, symmetry concerns
- **Várady et al. (2016)**
  - Generalized Bézier patches
  - Regular polygonal domain
  - Symmetric control structure

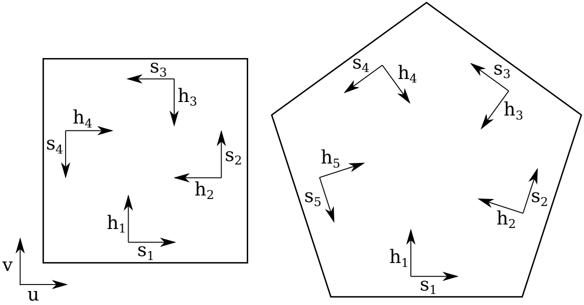
Control net derivation from the quadrilateral case

- Control grid → n ribbons
- Degree: d
- Layers: $l = \left\lfloor \frac{d+1}{2} \right\rfloor$
- Control points:
  - $C^{d,i}_{j,k}$
  - $i = 1 \dots n$
  - $j = 0 \dots d$
  - $k = 0 \dots l-1$
- Weights: $\mu^i_{j,k}$



Domain

- Regular domain in the (u, v) plane
- Side-based local parameterization functions s_i and h_i
  - Based on Wachspress barycentric coordinates $\lambda_i(u, v)$

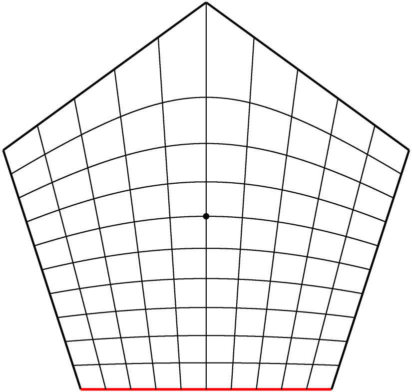


Local parameters

- $s_i = \frac{\lambda_i}{\lambda_{i-1}+\lambda_i}$
- $h_i = 1 - \lambda_{i-1} - \lambda_i$

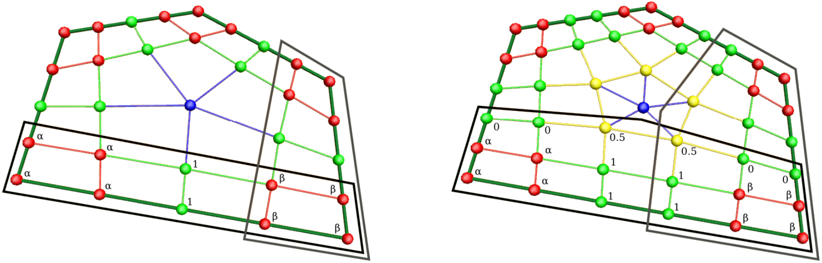
Barycentric coordinates λ_i

- $\lambda_i \geq 0$
  [positivity]
- $\sum_{i=1}^{n} \lambda_i = 1$
  [partition of unity]
- $\sum_{i=1}^{n} \lambda_i(u,v) \cdot P_i = (u,v)$
  [reproduction]
- $\lambda_i(P_j) = \delta_{ij}$
  [Lagrange property]



Bernstein functions with rational weights

- $C^{d,i}_{j,k}$: j-th control point on side i, layer k
- Multiplied by $\mu^i_{j,k} B^d_{j,k}(s_i, h_i) = \mu^i_{j,k} B^d_j(s_i) B^d_k(h_i)$
- $\mu^i_{j,k}$ is a rational function for 2×2 CPs in each corner
- $\alpha_i = h_{i-1}/(h_{i-1} + h_i)$, $\beta_i = h_{i+1}/(h_{i+1} + h_i)$



Central weight & patch equation

- Weights do not add up to 1
- Deficiency ⇒ weight of the central point:

$$B_0^d(u,v) = 1 - \sum_{i=1}^{n}\sum_{j=0}^{d}\sum_{k=0}^{l-1}\mu_{j,k}^i B_{j,k}^d(s_i,h_i)$$

- Patch equation:

$$S(u,v) = \sum_{i=1}^{n}\sum_{j=0}^{d}\sum_{k=0}^{l-1} C_{j,k}^{d,i}\mu_{j,k}^i B_{j,k}^d(s_i,h_i) + C_0^d B_0^d(u,v)$$

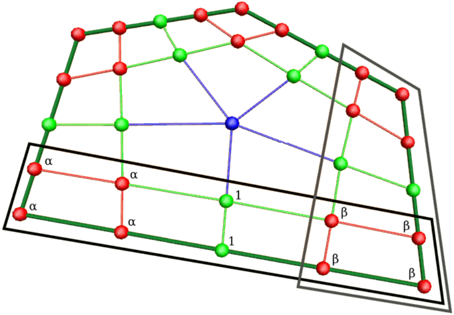
Interpolation property

Definition

A Bézier ribbon is a Bézier patch given by the first two layers (rows) of control points on a given side.

Theorem

The Generalized Bézier patch, on its boundary, interpolates the position and first cross-derivative of the Bézier ribbons of its respective sides.



Overview

Fixed issues

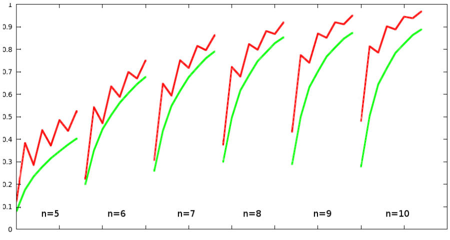
- Weight deficiency
  - Increases with n and d $\Rightarrow$
    Influence of the central control point grows
  - Strongly oscillates between even and odd degrees
- Support for G^2 continuity between patches

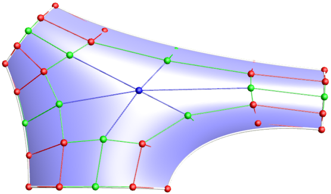
New/updated algorithms

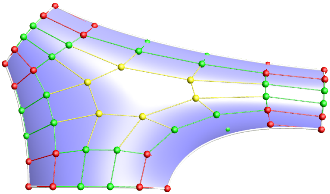
- Degree elevation & reduction
- Fullness control
- Approximation of point clouds

Weight deficiency

- No central control point for odd-degree patches
- For even-degree patches: $C^d_{l,l} \cdot \sum_{i=1}^{n} \mu^i_{l,l} B^d_{l,l}(s_i, h_i)$
- Weight deficiency is distributed amongst the innermost blend functions

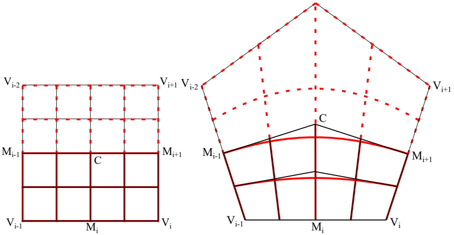


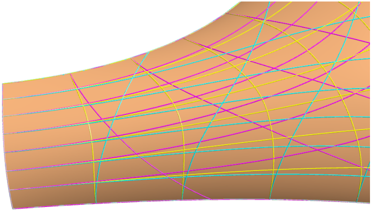


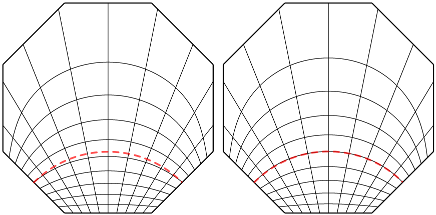


New parameterization

- Better isoline distribution
- Lower weight deficiency
- Constraints:
  - $h_i = 0.5$ tangential to s_{i-1} and s_{i+1}
  - Middle point on a circular arc



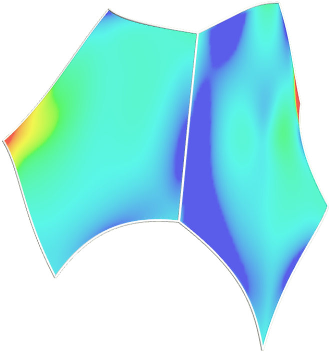


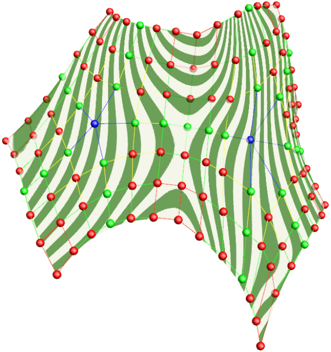


G^2 continuity

- Use squared terms in the rational weights:

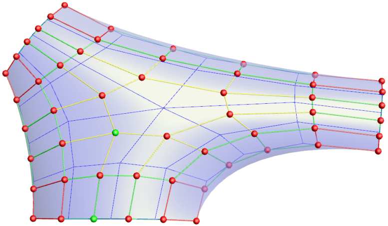
$$\alpha_i = h_{i-1}^2/(h_{i-1}^2 + h_i^2) \qquad \beta_i = h_{i+1}^2/(h_{i+1}^2 + h_i^2)$$

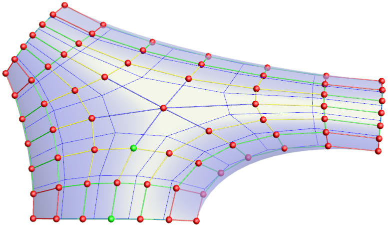




Degree elevation & reduction

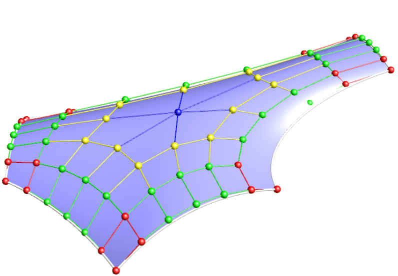
- Essentially the same as in the original paper
- Linear and bilinear combinations
- Modifies the surface (slightly)
- The control net is generated by reductions and elevations
  - Default positions for internal control points





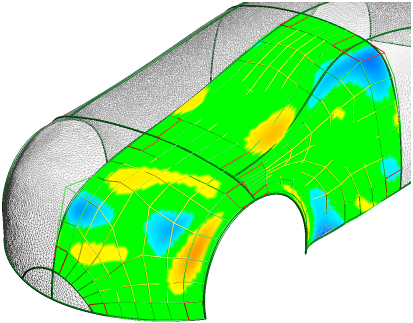
Fullness control

- Multi-resolution editing technique
- Edit a control point of a lower-degree patch
  - E.g. quartic central point
- Its influence is propagated by degree elevation

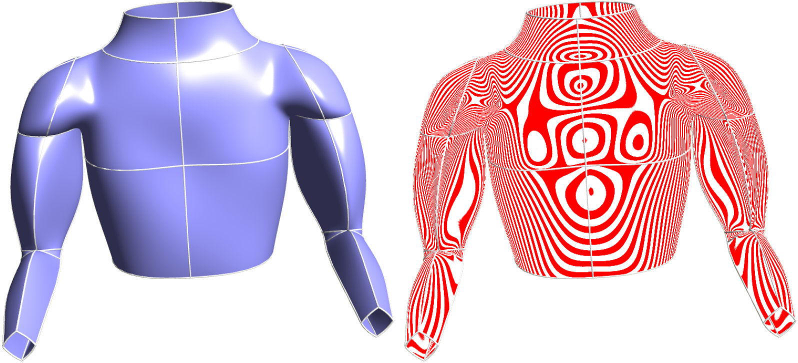


Approximation

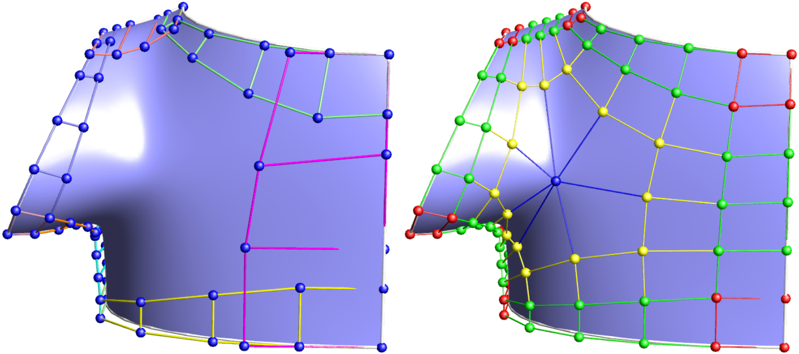
- Least-squares fit of points
- Initial surface
  - Generated by the boundary constraints
- Initial parameterization
  - Projection
- Iteration:
  - Fit with smoothing
  - Degree elevation
  - Re-parameterization
- Smoothing
  - Reduce oscillation of the control points



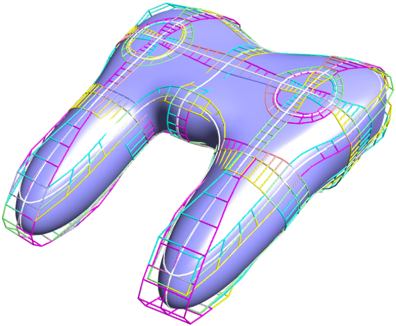
Torso

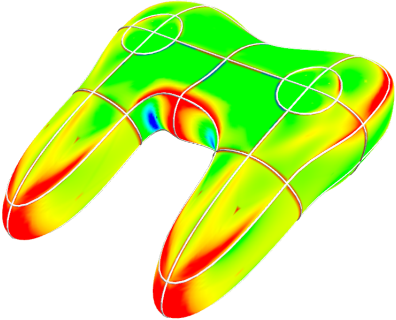


Torso – detail

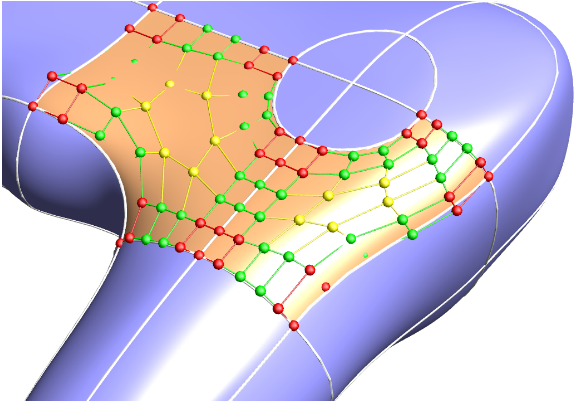


Gamepad





Gamepad – detail



Summary

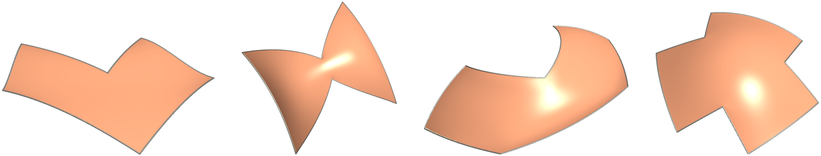
Generalized Bézier patches

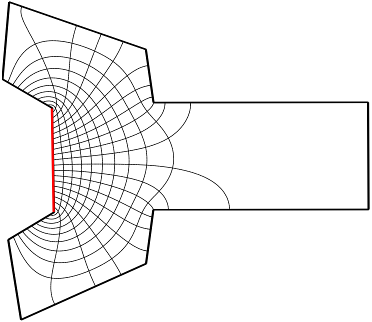
- Side-based interpretation
- All control points generated by the boundaries via degree elevation
- Interior control

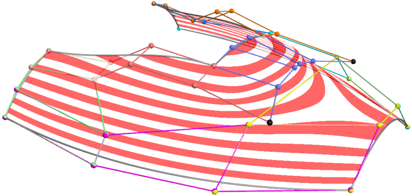
Enhancements

- Follow the quadrilateral patch more closely
- Central control point / weight deficiency fixed
- Better parameterization
- Curvature continuity
- Approximation algorithm

Patches over concave domains







Any questions?

Thank you for your attention.