Enhancement of a multi-sided Bézier surface representation

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Outline

1. Introduction
   - Motivation
   - Previous work

2. Generalized Bézier (GB) patch
   - Control structure
   - Domain & parameterization
   - Blending functions

3. Enhancements
   - Problems
   - New algorithms

4. Examples

5. Conclusion and future work
Applications of multi-sided patches

- Curve network based design
  - Feature curves
  - Automatic surface generation
- Hole filling
  - E.g. vertex blends
  - Cross-derivative constraints
- 3D point cloud approximation
  - Given boundary loops
  - Smoothly connected patches

Representation?
Conventional representations

- Trimmed/split tensor product surfaces
  - Detailed control in the interior
  - CAD-compatible
  - But: continuity problems

- Recursive subdivision
  - Arbitrary topology
  - Easy to design with
  - But: hard to interpolate boundary cross-derivatives

- Transfinite patches
  - Interpolates any number of sides
  - Depends only on the boundary
  - But: little interior control
Multi-sided surfaces with control networks

- Loop and DeRose (1989)
  - S-patches – beautiful theory, difficult to use
- Warren (1992)
  - Based on Bézier triangles, max. 6 sides
- Zheng and Ball (1997)
  - High-degree expressions, max. 6 sides
- Krasauskas (2002)
  - Toric patches – lattice-based, symmetry concerns
- Várady et al. (2016)
  - Generalized Bézier patches
  - Regular polygonal domain
  - Symmetric control structure
Control net derivation from the quadrilateral case

- Control grid $\rightarrow$ $n$ ribbons
- Degree: $d$
- Layers: $l = \left\lfloor \frac{d+1}{2} \right\rfloor$
- Control points: $C_{j,k}^{d,i}$
  - $i = 1 \ldots n$
  - $j = 0 \ldots d$
  - $k = 0 \ldots l - 1$
- Weights: $\mu_{j,k}^i$
Domain

- Regular domain in the \((u, v)\) plane
- Side-based local parameterization functions \(s_i\) and \(h_i\)
  - Based on Wachspress barycentric coordinates \(\lambda_i(u, v)\)
Local parameters

- $s_i = \frac{\lambda_i}{\lambda_{i-1} + \lambda_i}$
- $h_i = 1 - \lambda_{i-1} - \lambda_i$

Barycentric coordinates $\lambda_i$

- $\lambda_i \geq 0$ [positivity]
- $\sum_{i=1}^{n} \lambda_i = 1$ [partition of unity]
- $\sum_{i=1}^{n} \lambda_i(u, v) \cdot P_i = (u, v)$ [reproduction]
- $\lambda_i(P_j) = \delta_{ij}$ [Lagrange property]
Bernstein functions with rational weights

- \( C_{j,k}^{d,i} \): j-th control point on side i, layer k
- Multiplied by \( \mu_{j,k}^i B_{j,k}^d(s_i, h_i) = \mu_{j,k}^i B_j^d(s_i)B_k^d(h_i) \)
- \( \mu_{j,k}^i \) is a rational function for 2 × 2 CPs in each corner
- \( \alpha_i = h_{i-1}/(h_{i-1} + h_i), \beta_i = h_{i+1}/(h_{i+1} + h_i) \)
Central weight & patch equation

- Weights do not add up to 1
- Deficiency $\Rightarrow$ weight of the central point:

$$B^d_0(u, v) = 1 - \sum_{i=1}^{n} \sum_{j=0}^{d} \sum_{k=0}^{l-1} \mu_{j,k} B^d_{j,k}(s_i, h_i)$$

- Patch equation:

$$S(u, v) = \sum_{i=1}^{n} \sum_{j=0}^{d} \sum_{k=0}^{l-1} C^d_{j,k} \mu_{j,k} B^d_{j,k}(s_i, h_i) + C^d_0 B^d_0(u, v)$$
Interpolation property

**Definition**
A Bézier ribbon is a Bézier patch given by the first two layers (rows) of control points on a given side.

**Theorem**
The Generalized Bézier patch, on its boundary, interpolates the position and first cross-derivative of the Bézier ribbons of its respective sides.
Overview

Fixed issues
- Weight deficiency
  - Increases with \( n \) and \( d \) \( \Rightarrow \)
    - Influence of the central control point grows
  - Strongly oscillates between even and odd degrees

- Support for \( G^2 \) continuity between patches

New/updated algorithms
- Degree elevation & reduction
- Fullness control
- Approximation of point clouds
Weight deficiency

- No central control point for odd-degree patches
- For even-degree patches:
  \[ C_{d,l} \cdot \sum_{i=1}^{n} \mu_{i,l} B_{l,l}^{d}(s_i, h_i) \]
- Weight deficiency is distributed amongst the innermost blend functions
New parameterization

- Better isoline distribution
- Lower weight deficiency
- Constraints:
  - $h_i = 0.5$ tangential to $s_{i-1}$ and $s_{i+1}$
  - Middle point on a circular arc
Use squared terms in the rational weights:

\[ \alpha_i = \frac{h_i^2}{h_{i-1}^2 + h_i^2} \quad \beta_i = \frac{h_{i+1}^2}{h_{i+1}^2 + h_i^2} \]
Degree elevation & reduction

- Essentially the same as in the original paper
- Linear and bilinear combinations
- Modifies the surface (slightly)
- The control net is generated by reductions and elevations
  - Default positions for internal control points
Fullness control

- Multi-resolution editing technique
- Edit a control point of a lower-degree patch
  - E.g. quartic central point
- Its influence is propagated by degree elevation
Approximation

- Least-squares fit of points
- Initial surface
  - Generated by the boundary constraints
- Initial parameterization
  - Projection
- Iteration:
  - Fit with smoothing
  - Degree elevation
  - Re-parameterization
- Smoothing
  - Reduce oscillation of the control points
Example 1

Torso

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Example 1

Torso – detail

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Enhancement of a multi-sided Bézier surface representation
Gamepad

Enhancement of a multi-sided Bézier surface representation
Example 2

Gamepad – detail
Summary

**Generalized Bézier patches**
- Side-based interpretation
- All control points generated by the boundaries via degree elevation
- Interior control

**Enhancements**
- Follow the quadrilateral patch more closely
- Central control point / weight deficiency fixed
- Better parameterization
- Curvature continuity
- Approximation algorithm
Patches over concave domains
Any questions?

Thank you for your attention.