Multi-sided surfaces interpolating arbitrary boundaries with intuitive interior control

Péter Salvi

Budapest University of Technology and Economics

Curves & Surfaces

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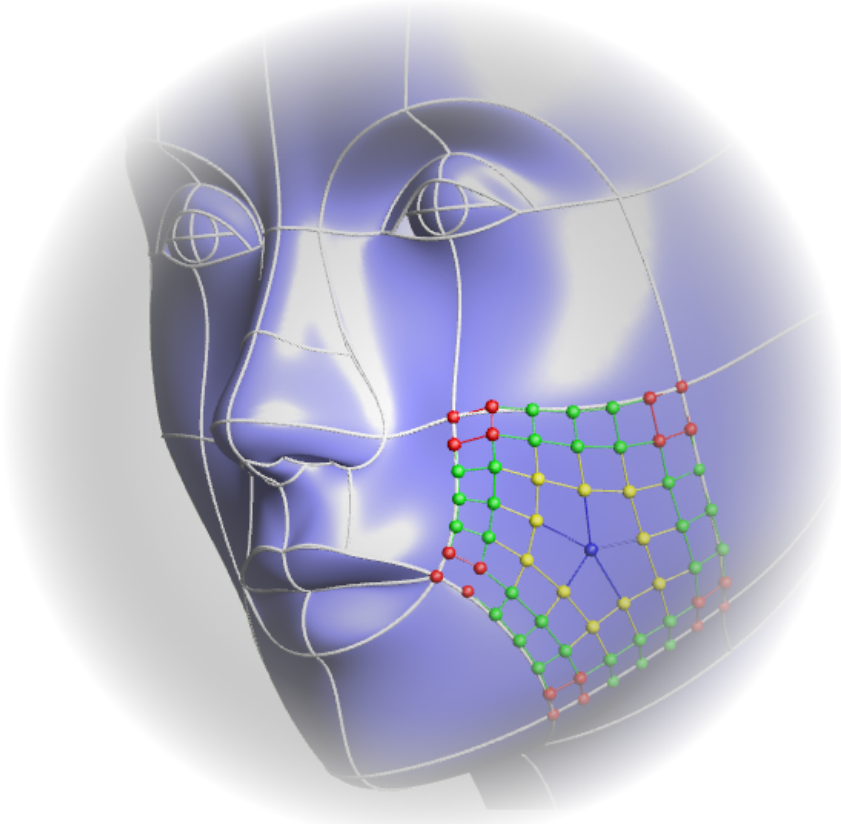
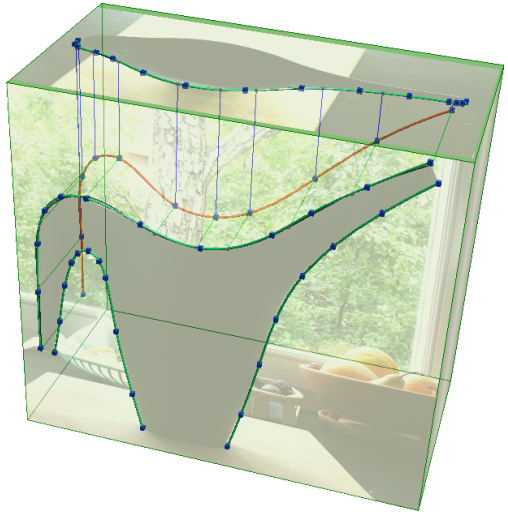
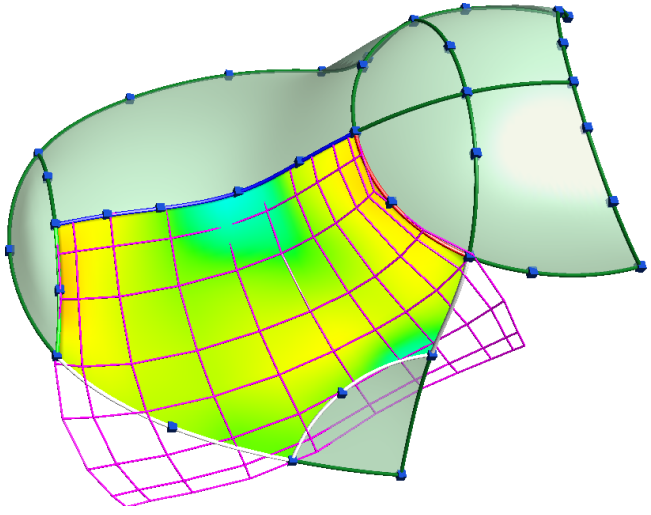
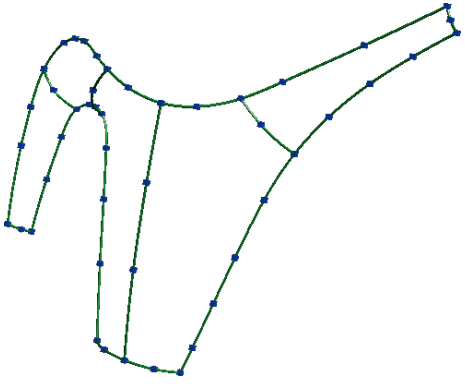
Outline

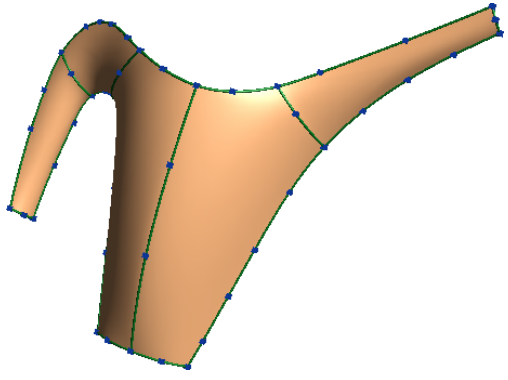


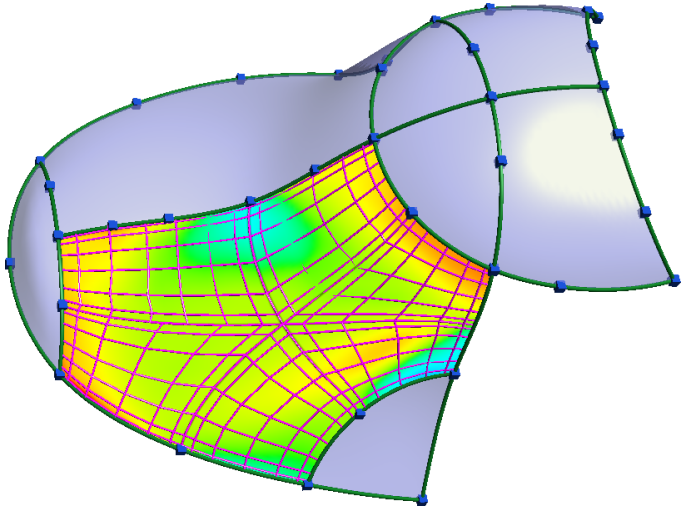
Motivation











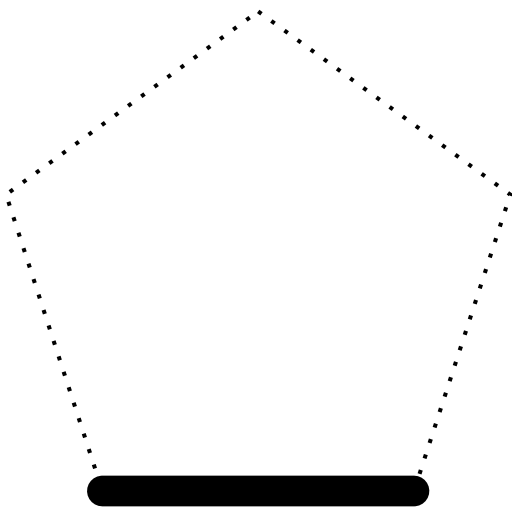
Outline

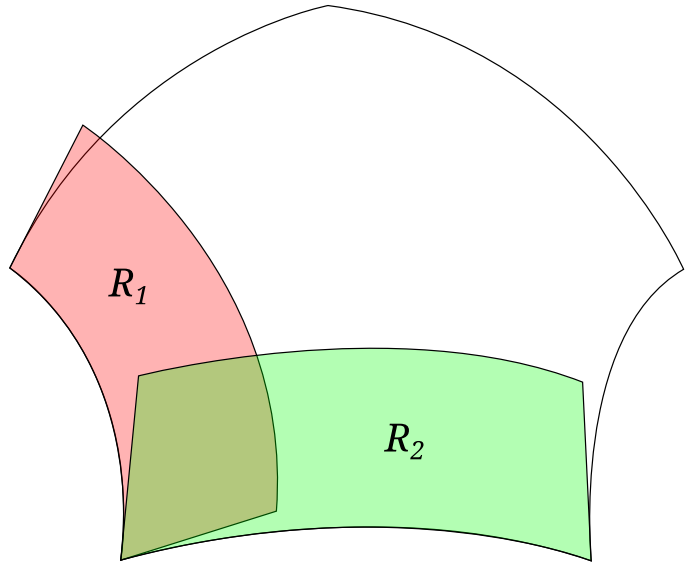
Katō's patch (CAD, 1991)

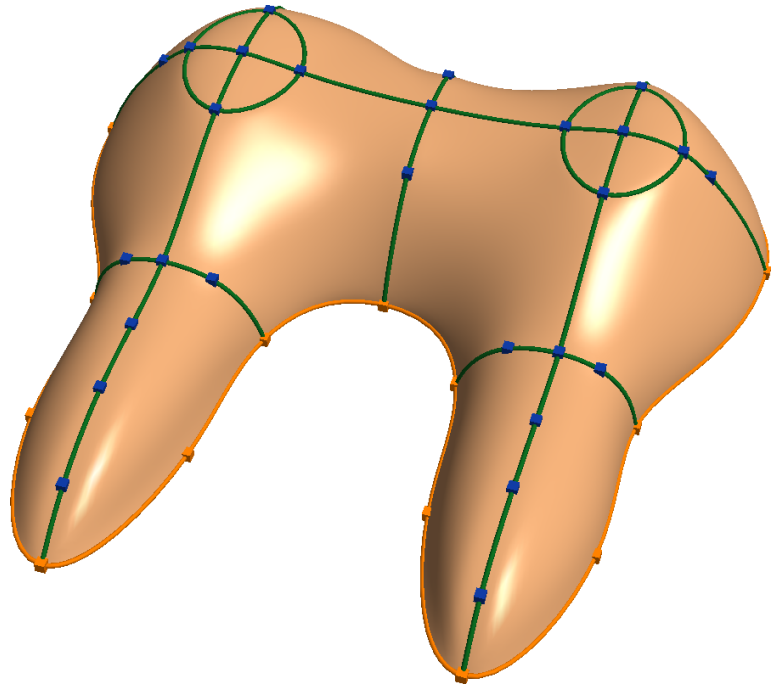
$$\mathbf{S}(u,v) = \sum_i \mathbf{R}_i(s_i, d_i) L_i(d_1, \ldots, d_n)$$

- Local parameters:
  side (s_i) & distance (d_i)
- Singular blending function:

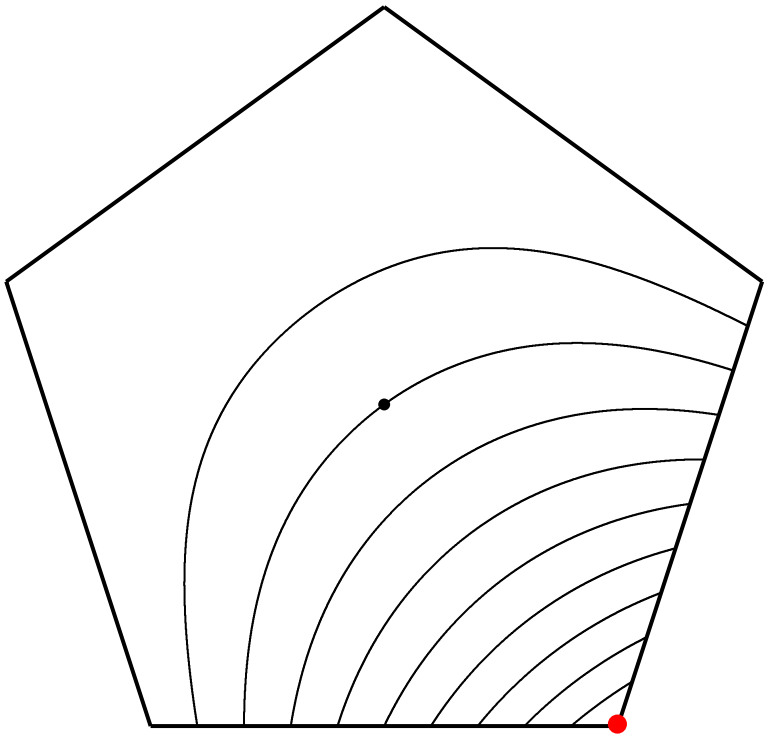
$$L_i(d_1, \ldots, d_n) = \frac{\prod_{k \neq i} d_k^2}{\sum_j \prod_{k \neq j} d_k^2}$$

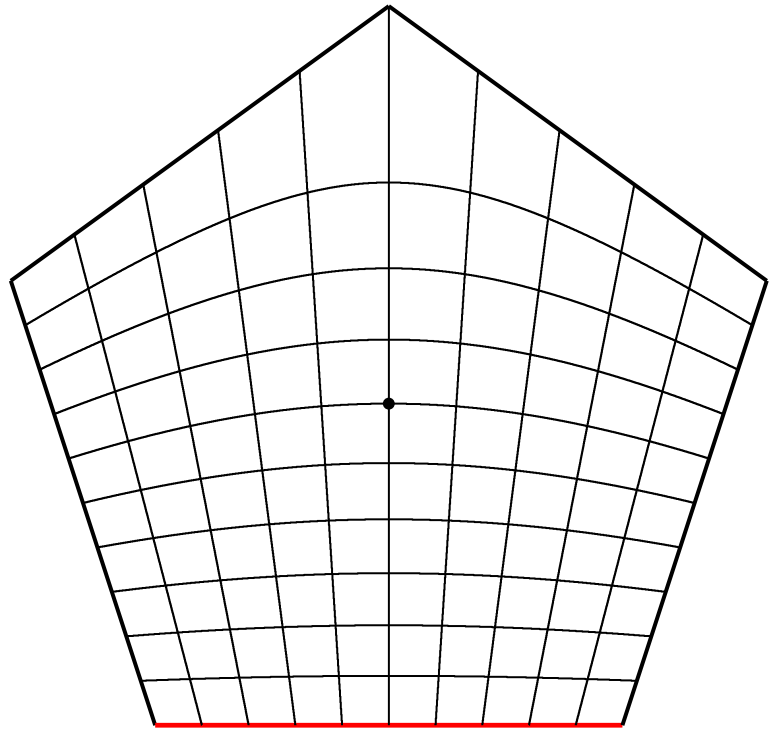






Parameterization based on Wachspress coordinates

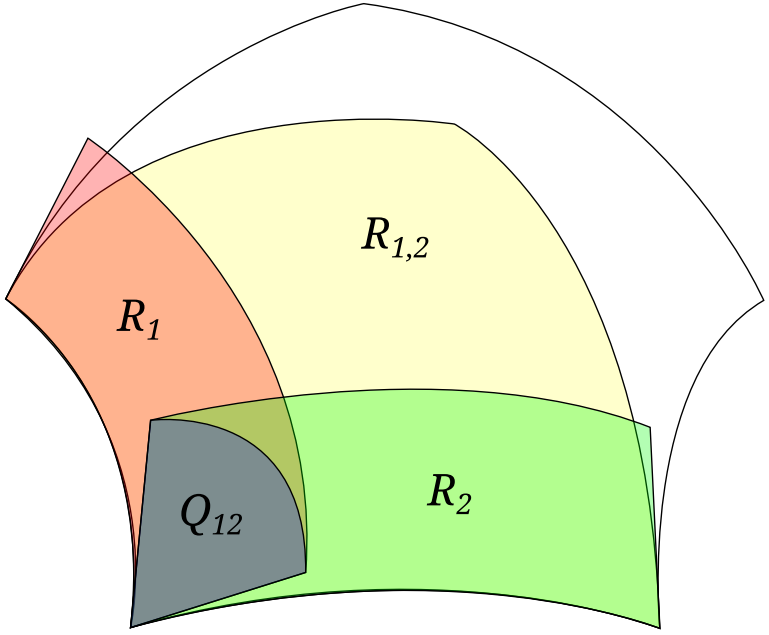


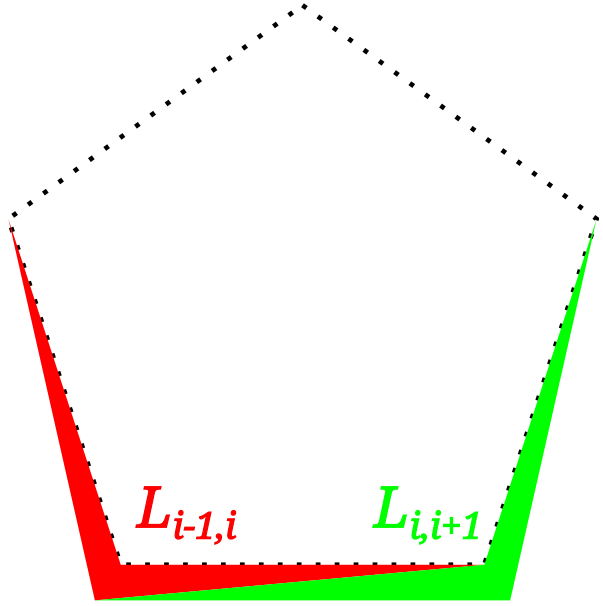


- side parameter $s_i = \lambda_i \,/\, (\lambda_{i-1} + \lambda_i)$
- distance parameter $d_i = 1 - (\lambda_{i-1} + \lambda_i)$

Charrot–Gregory patch (CAGD, 1984)

- Corner interpolant $\approx$ partial Coons patch
- $\mathbf{S} = \sum_i \mathbf{R}_{i-1,i} L_{i-1,i}$
- $\mathbf{R}_{i-1,i} = \mathbf{R}_{i-1} + \mathbf{R}_i - \mathbf{Q}_{i-1,i}$
- $L_{i-1,i} = \dfrac{\prod_{k \notin \{i,i-1\}} d_k^2}{\sum_j \prod_{k \notin \{j,j-1\}} d_k^2}$



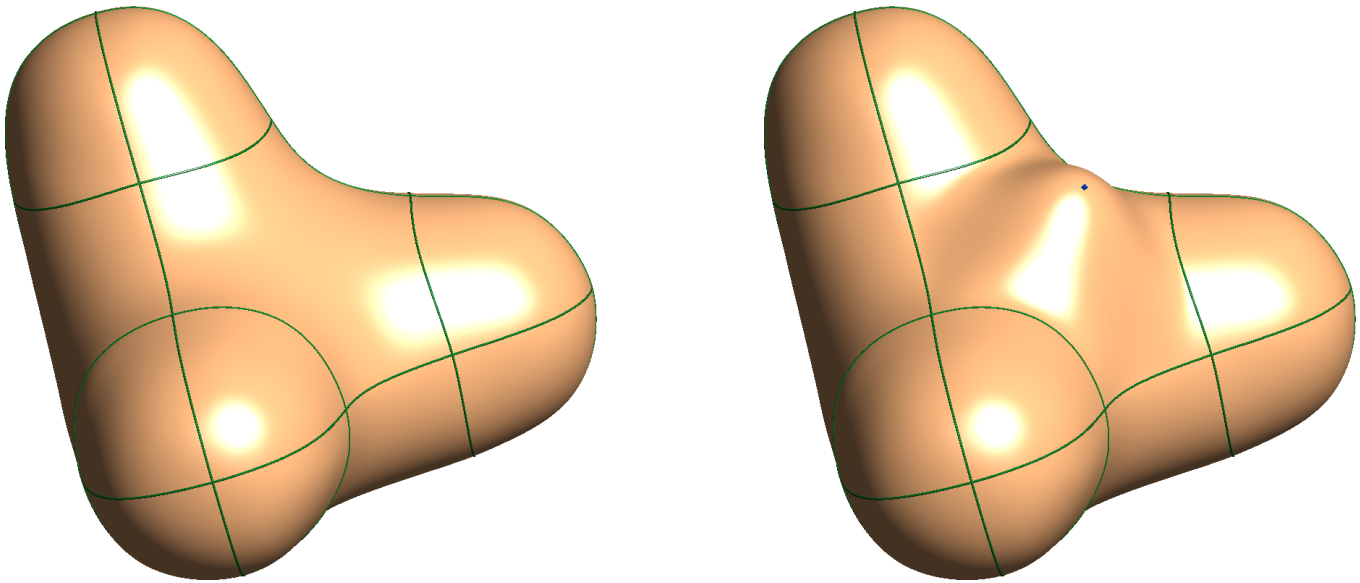


Midpoint patch

- Alternative blending function:

$$L^M_{i-1,i} = \frac{d_{i-1}\alpha_0(s_i)\alpha_0(d_i) + d_i\alpha_1(s_{i-1})\alpha_0(d_{i-1})}{d_{i-1} + d_i}$$

- $\alpha_0(x) = 1 - \alpha_1(x) = 2x^3 - 3x^2 + 1$ (Hermite blends)
- Weight deficient $\rightarrow$ extra DoF

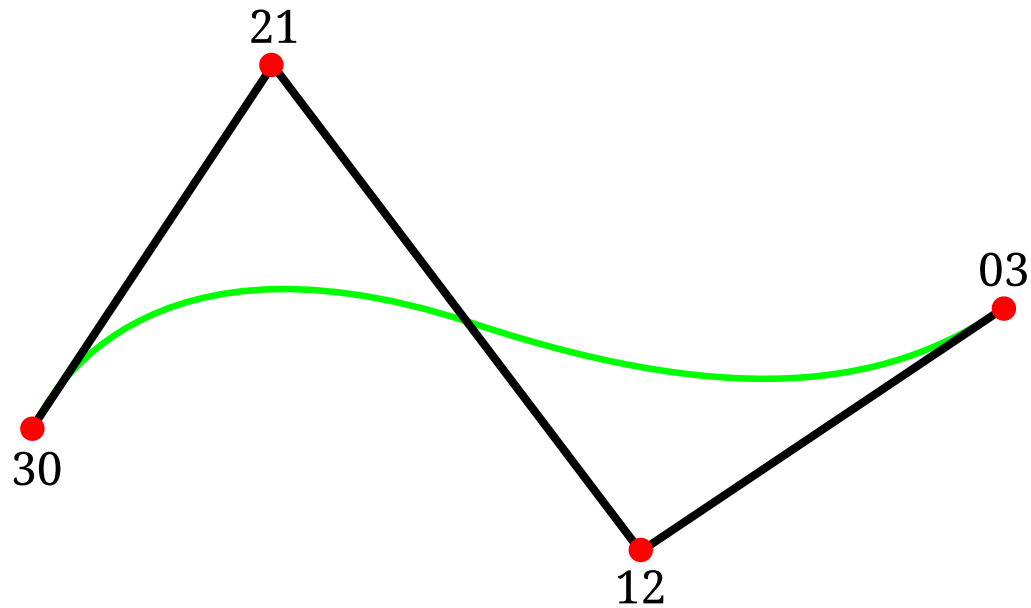


Outline

S-patch [Loop–DeRose] (ACM TOG, 1989)

$$\mathbf{C}(u) = \sum_{\mathbf{J}} \mathbf{P}_{\mathbf{J}} \frac{p!}{\prod_i J_i!} \prod_i \lambda_i^{J_i}$$

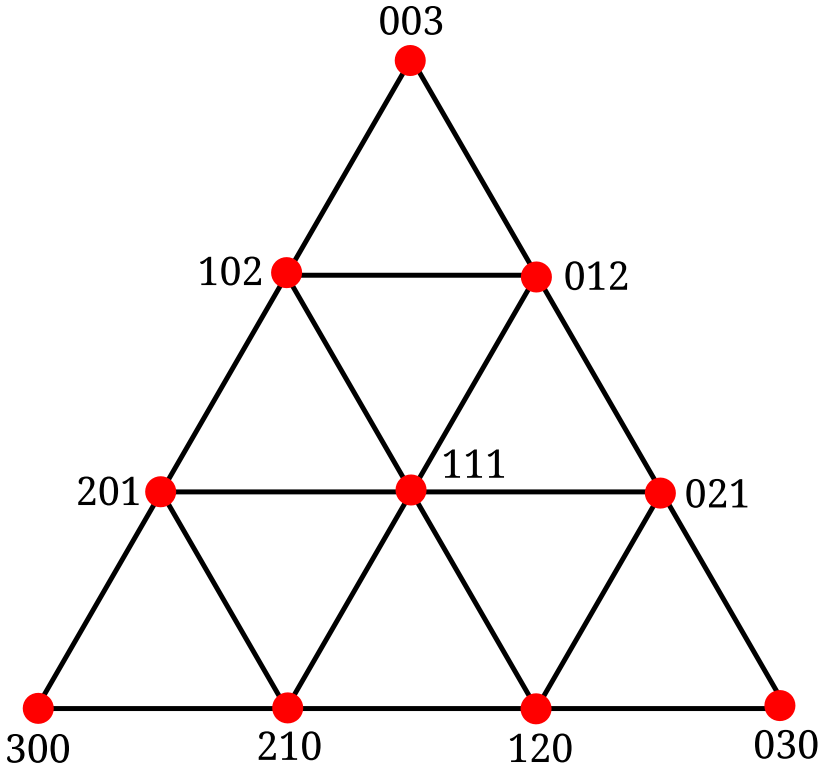
- $\mathbf{J}$ index vector
- $|\mathbf{J}| = 2$
- $\sum_i J_i = p$ (degree)
- $\lambda = (u, 1-u)$



S-patch [Loop–DeRose] (ACM TOG, 1989)

$$\mathbf{S}(u,v) = \sum_{\mathbf{J}} \mathbf{P}_{\mathbf{J}} \frac{p!}{\prod_i J_i!} \prod_i \lambda_i^{J_i}$$

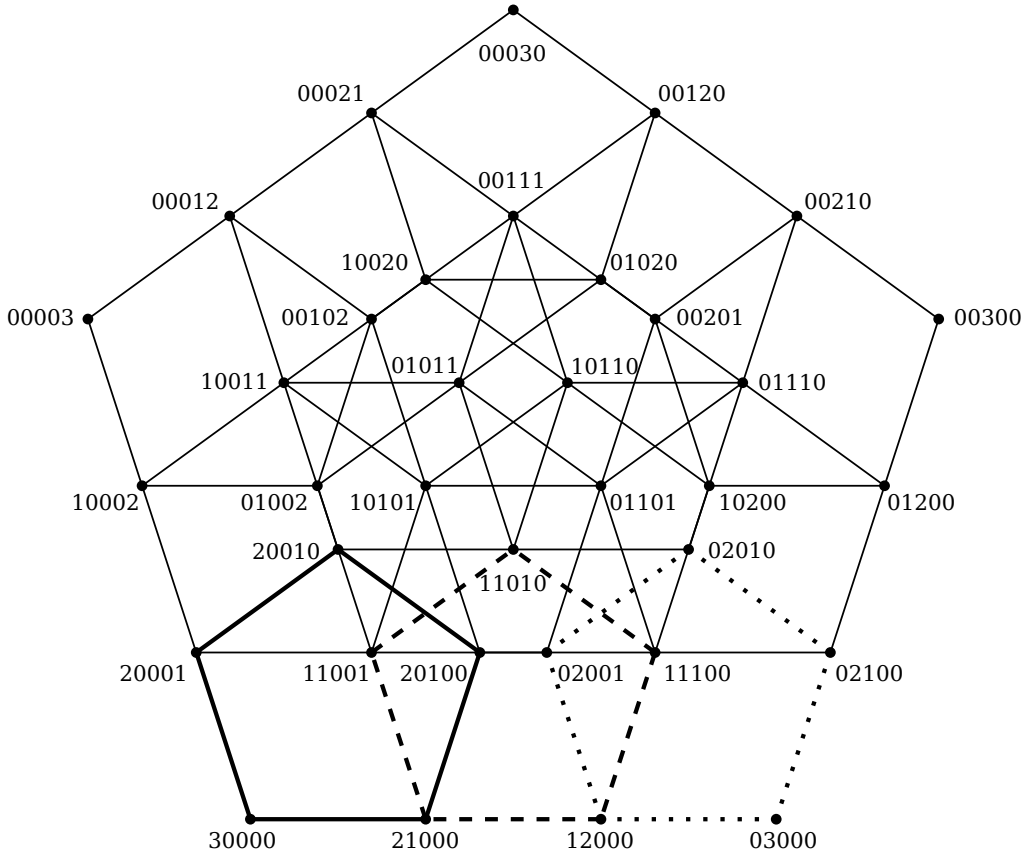
- $\mathbf{J}$ index vector
- $|\mathbf{J}| = 3$
- $\sum_i J_i = p$ (degree)
- λ: barycentric coords.



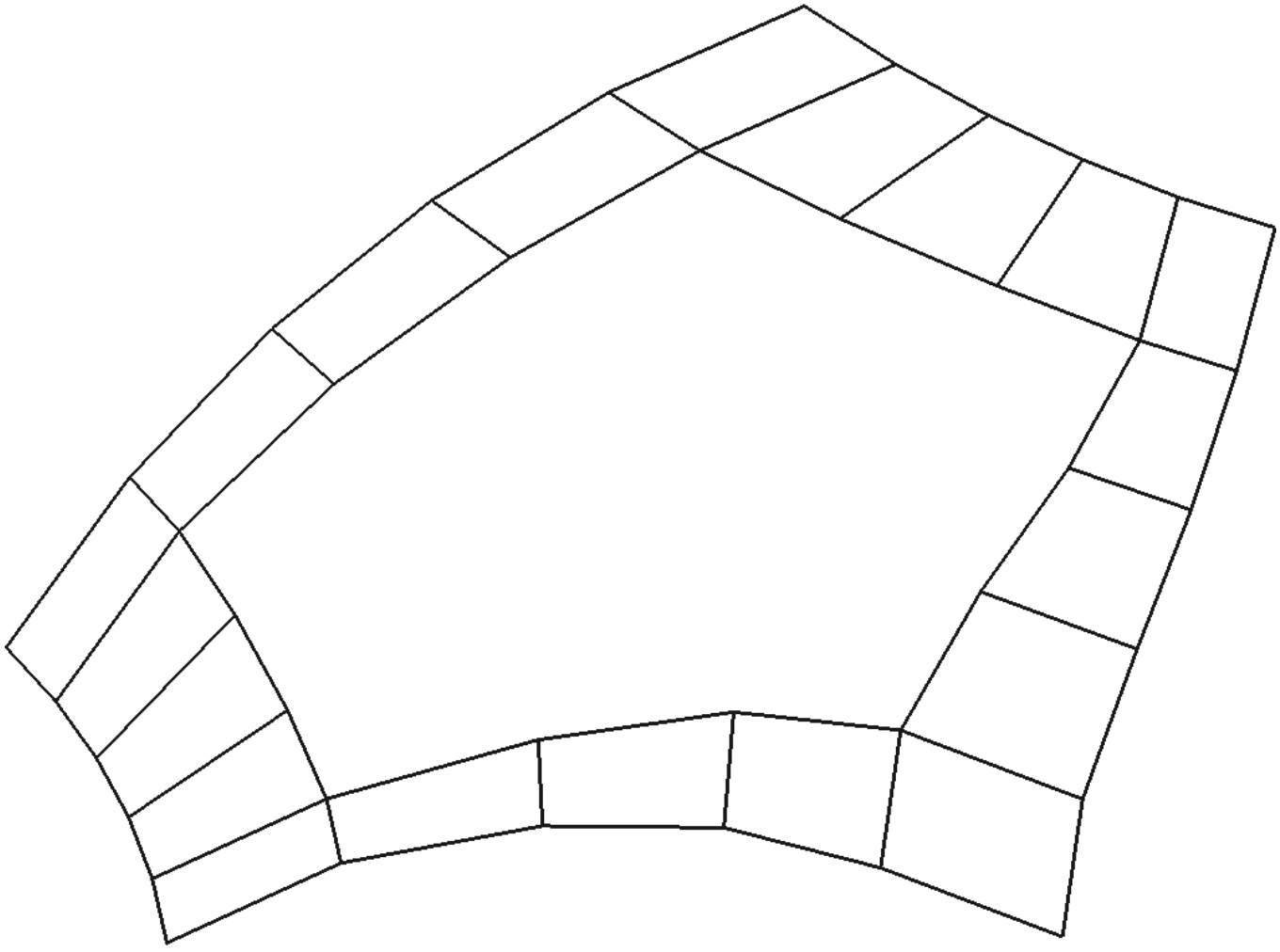
S-patch [Loop–DeRose] (ACM TOG, 1989)

$$\mathbf{S}(u,v) = \sum_{\mathbf{J}} \mathbf{P}_{\mathbf{J}} \frac{p!}{\prod_i J_i!} \prod_i \lambda_i^{J_i}$$

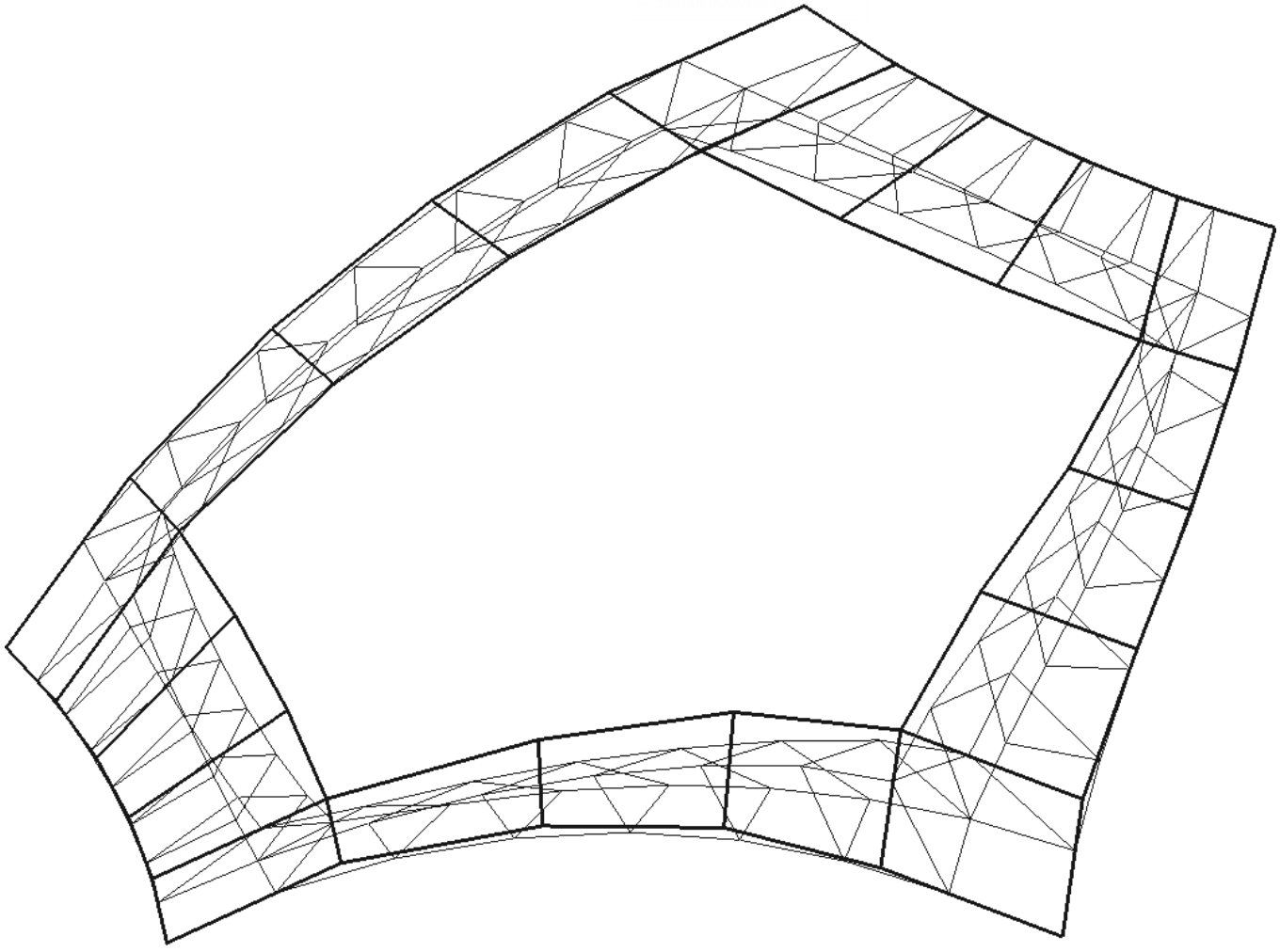
- $\mathbf{J}$ index vector
- $|\mathbf{J}| = n$
- $\sum_i J_i = p$ (depth)
- λ: GBC (Wachspress)



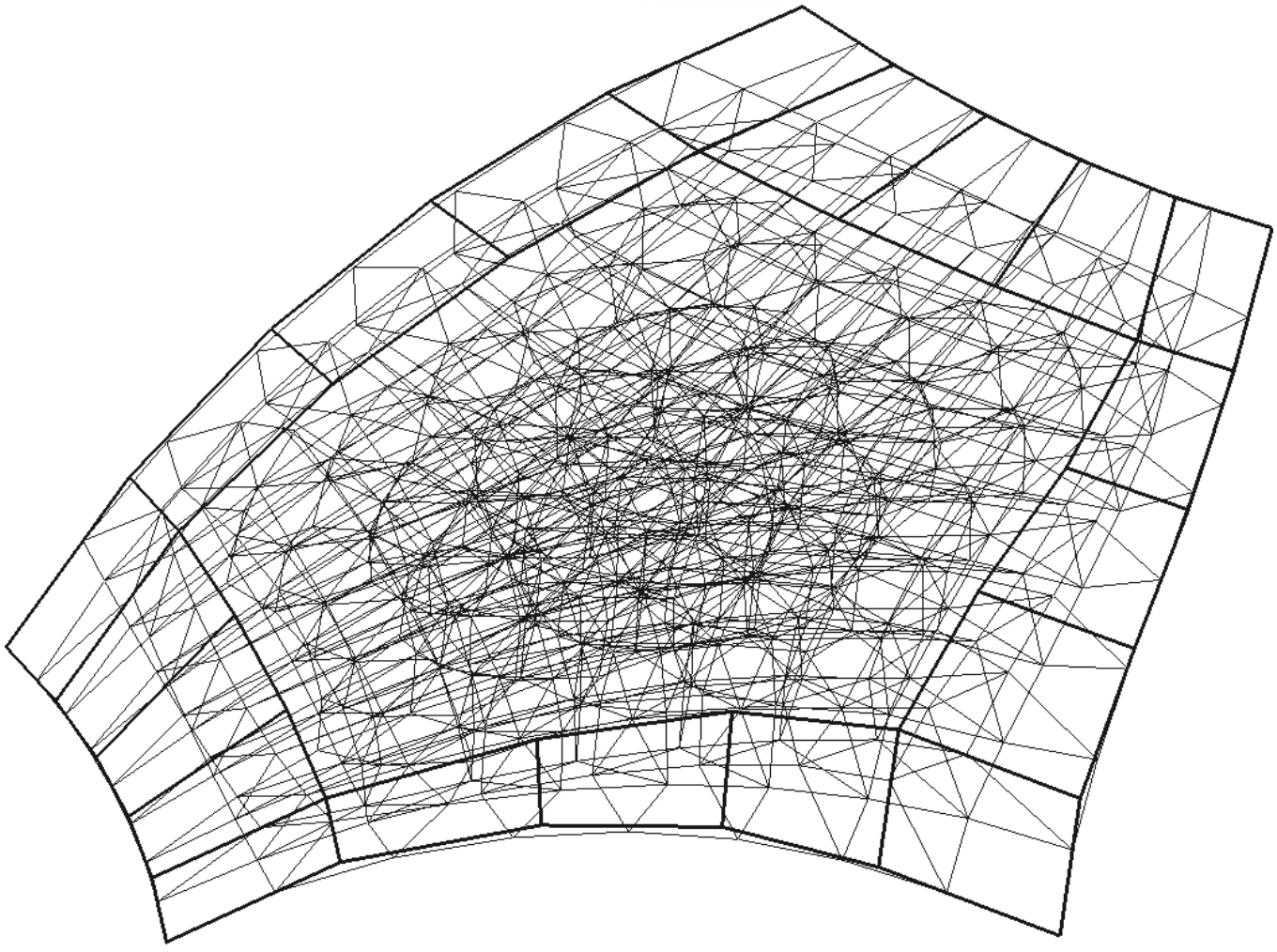
Transfinite interpolation with S-patches



Transfinite interpolation with S-patches



Transfinite interpolation with S-patches

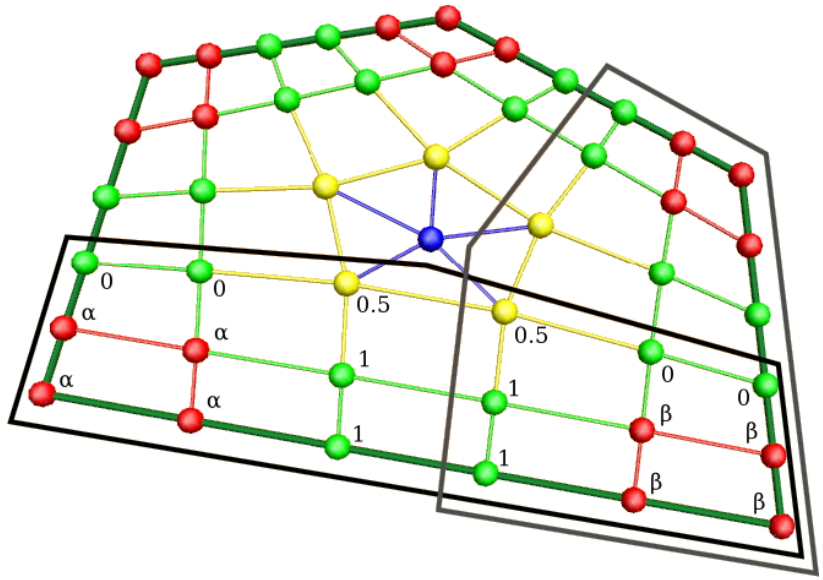


Generalized Bézier patch

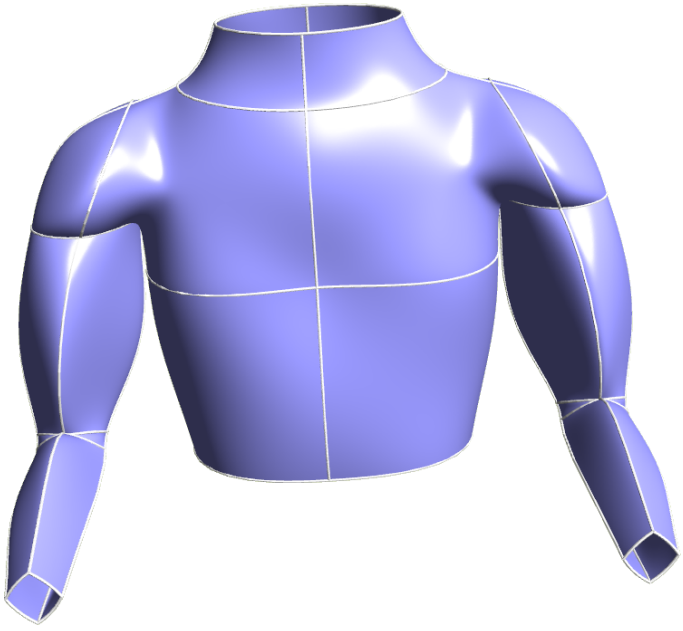
$$\mathbf{S}(u,v) = \sum_{i=1}^{n}\sum_{j=0}^{p}\sum_{k=0}^{\lfloor\frac{p-1}{2}\rfloor} \mathbf{C}_{i,j,k}\cdot\mu_{i,j,k}B_j^p(s_i)B_k^p(d_i)$$

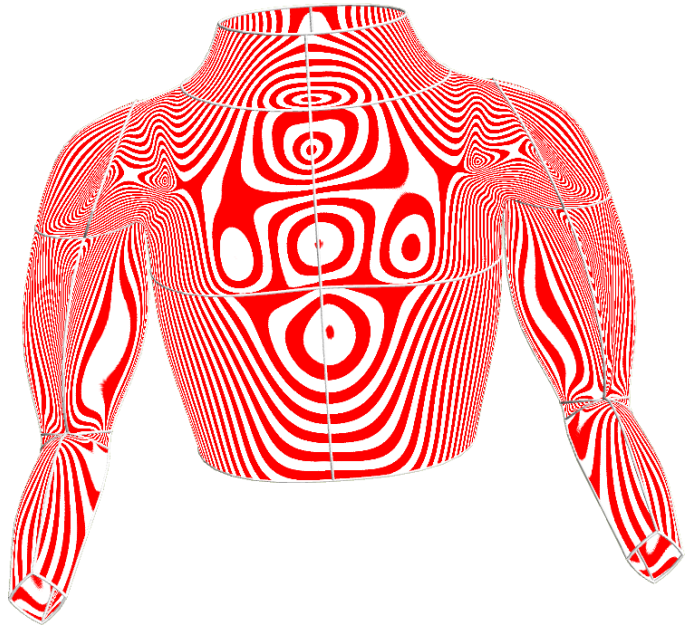
$$+\,\mathbf{C}_0\underbrace{\left(1-\sum_{i=1}^{n}\sum_{j=0}^{p}\sum_{k=0}^{\lfloor\frac{p-1}{2}\rfloor}\mu_{i,j,k}B_j^p(s_i)B_k^p(d_i)\right)}_{1-B_\Sigma(u,v)\text{ [weight deficiency]}}$$

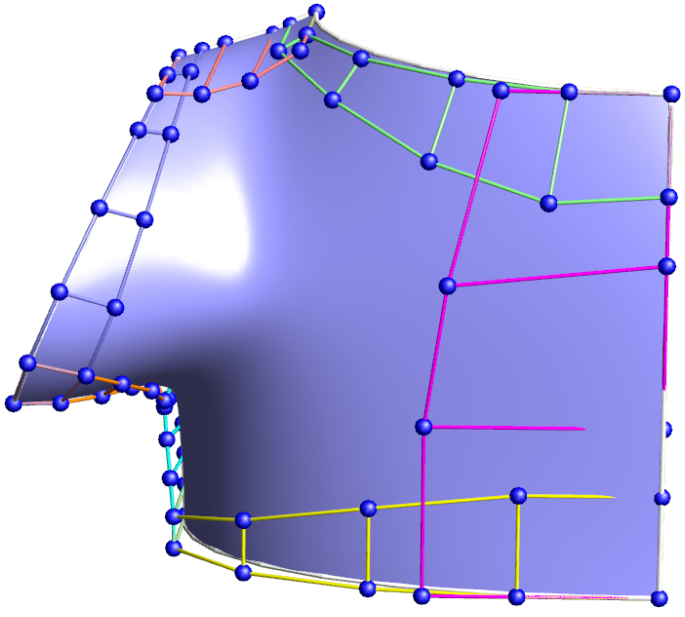
- $\mu_{i,j,k}$ rational weight
- $\alpha_i = d_{i-1}^2/(d_{i-1}^2+d_i^2)$
- $\beta_i = d_{i+1}^2/(d_{i+1}^2+d_i^2)$
- $\mathbf{C}_0$ central control

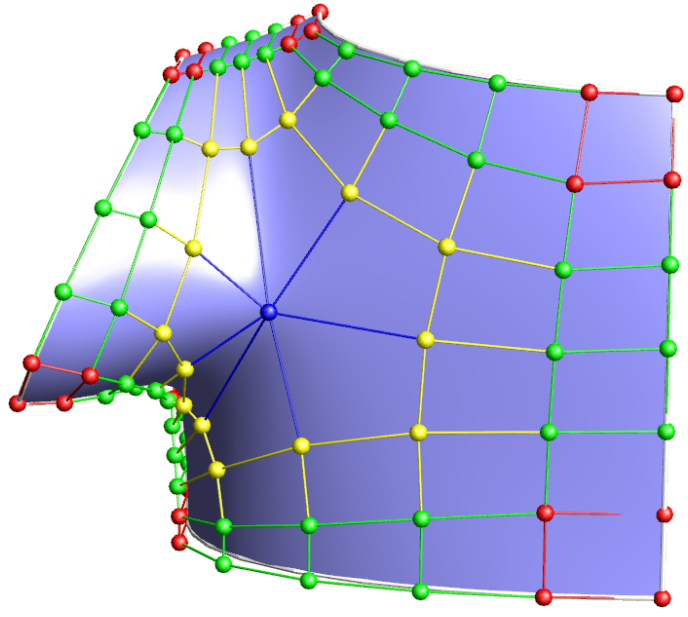


Example (shaded & isophote lines)

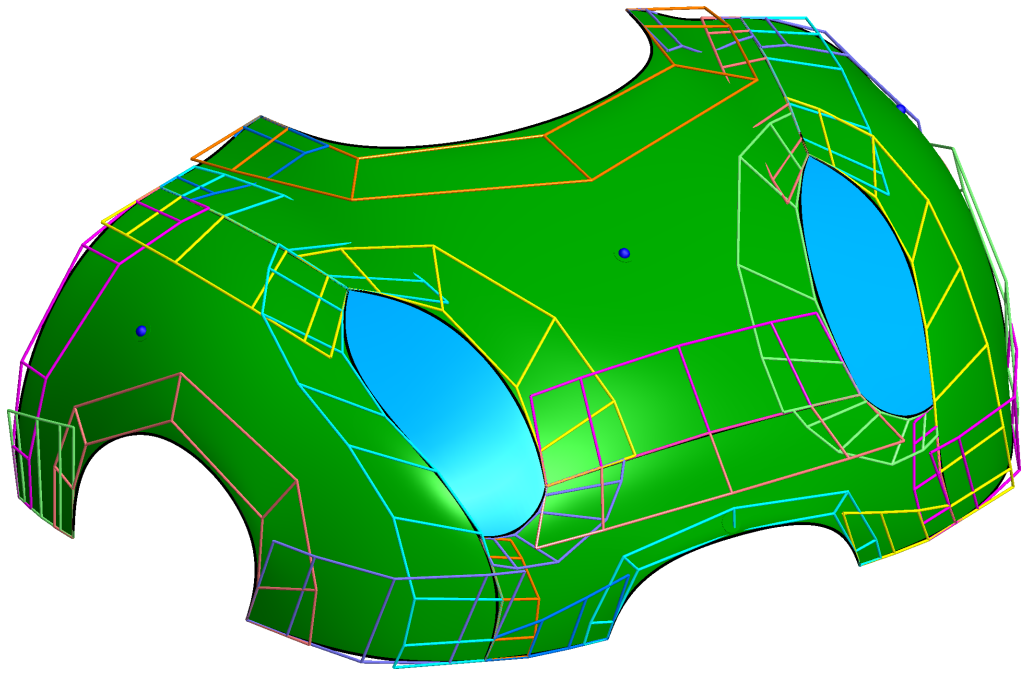


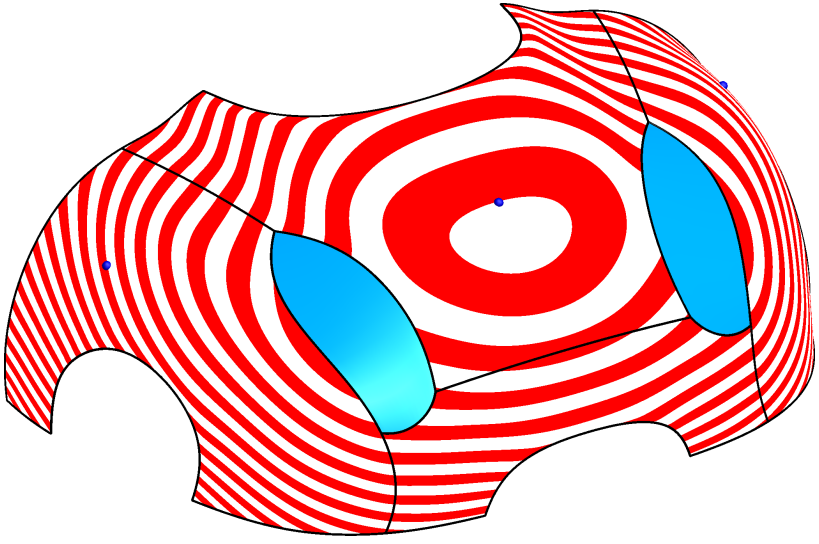


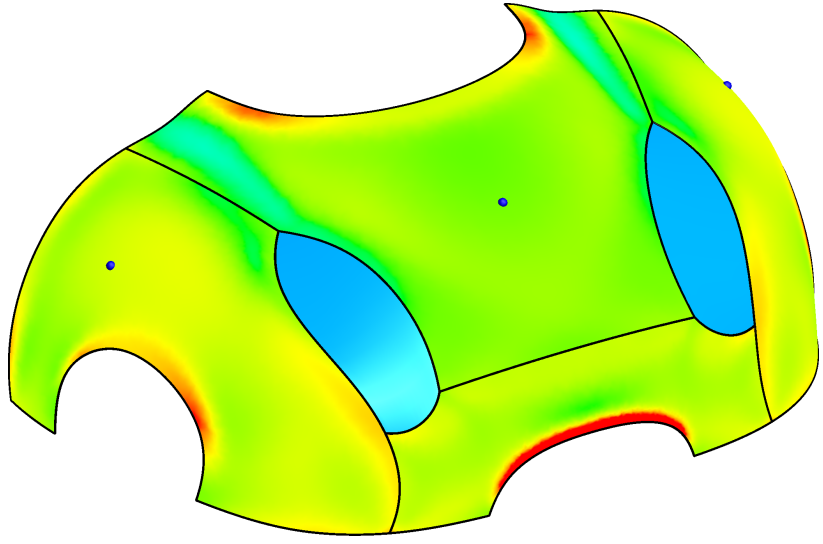




Generalized B-spline – (shaded, contouring & mean curv.)



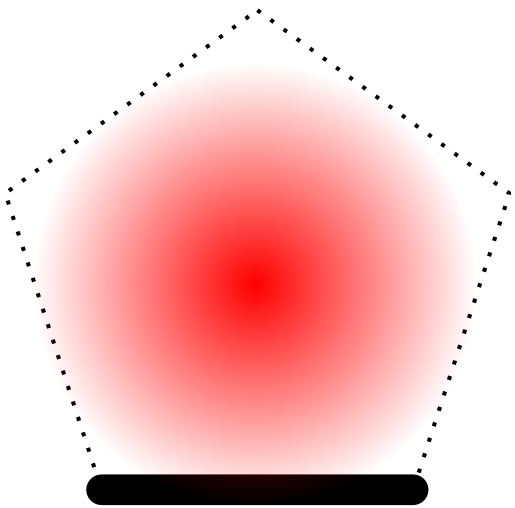


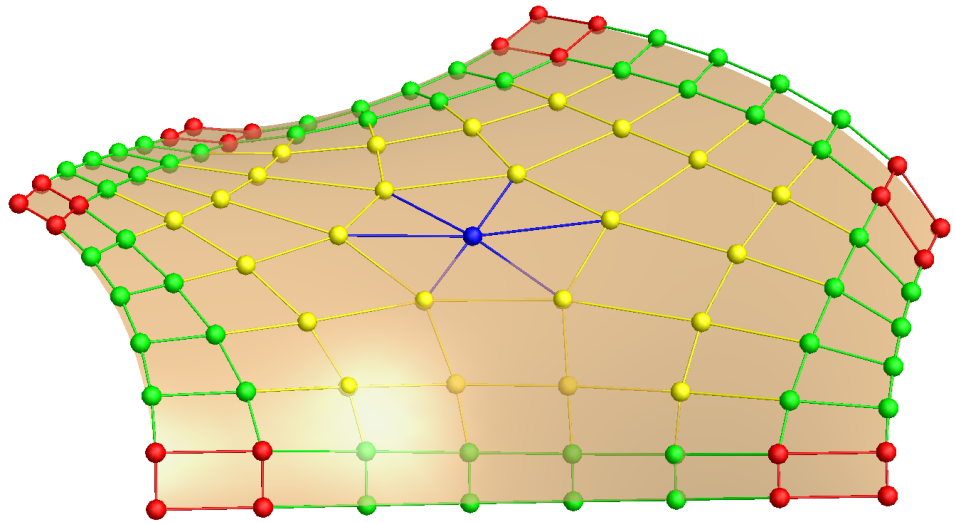


Outline

Hybrid patch

$$\mathbf{S}(u,v) = \sum_{i=1}^{n} \Bigg[\sum_{j=0}^{p} \sum_{k=2}^{\lfloor \frac{p-1}{2} \rfloor} \mathbf{C}_{i,j,k} \cdot \mu_{i,j,k} B_j^p(s_i) B_k^p(d_i) + \mathbf{R}_i(s_i,d_i) \cdot \underbrace{\sum_{j=0}^{p} \sum_{k=0}^{1} \mu_{i,j,k} B_j^p(s_i) B_k^p(d_i)}_{\text{similar to } L_i(d_1,\ldots,d_n)} \Bigg] + \mathbf{C}_0 B_\Sigma(u,v)$$

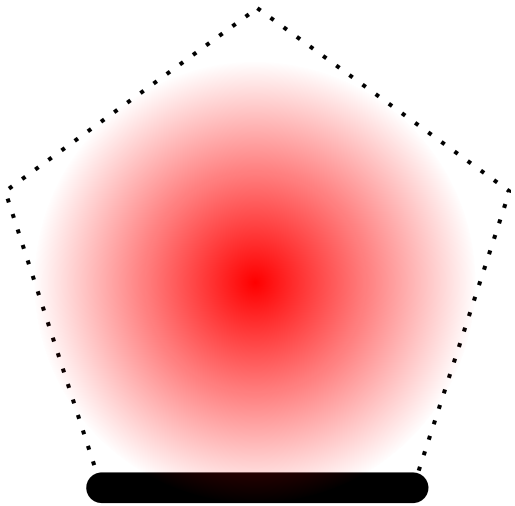


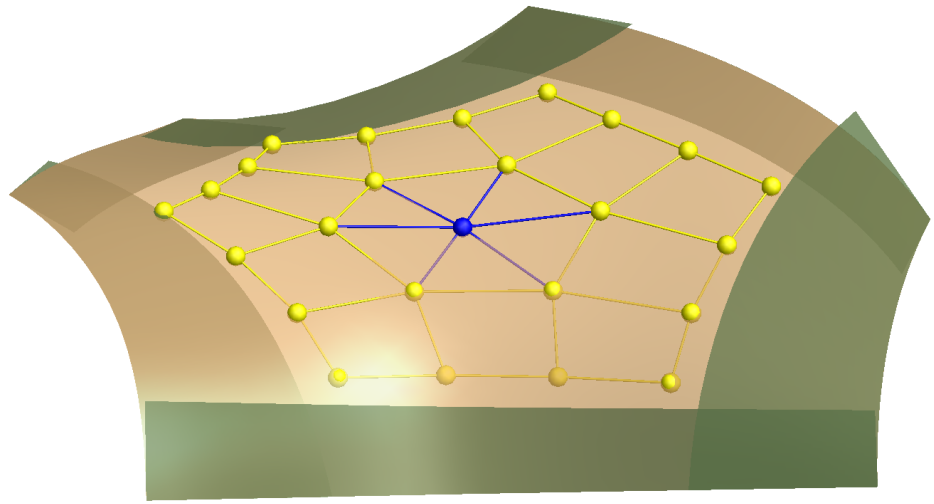


Hybrid patch

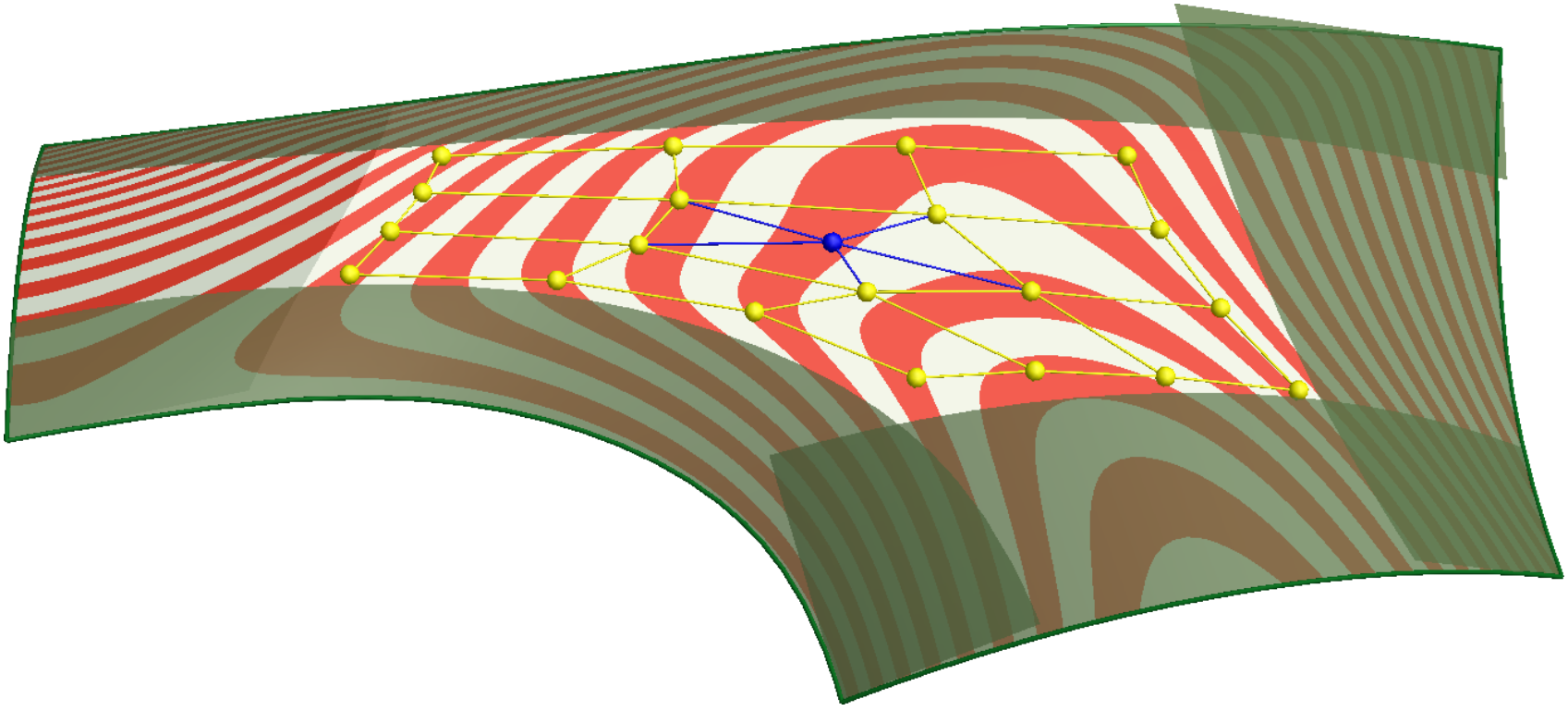
$$\mathbf{S}(u,v) = \sum_{i=1}^{n} \left[\sum_{j=0}^{p} \sum_{k=2}^{\lfloor \frac{p-1}{2} \rfloor} \mathbf{C}_{i,j,k} \cdot \mu_{i,j,k} B_j^p(s_i) B_k^p(d_i) \right.$$

$$\left. + \mathbf{R}_i(s_i, d_i) \cdot \underbrace{\sum_{j=0}^{p} \sum_{k=0}^{1} \mu_{i,j,k} B_j^p(s_i) B_k^p(d_i)}_{\text{similar to } L_i(d_1,\ldots,d_n)} \right] + \mathbf{C}_0 B_\Sigma(u,v)$$

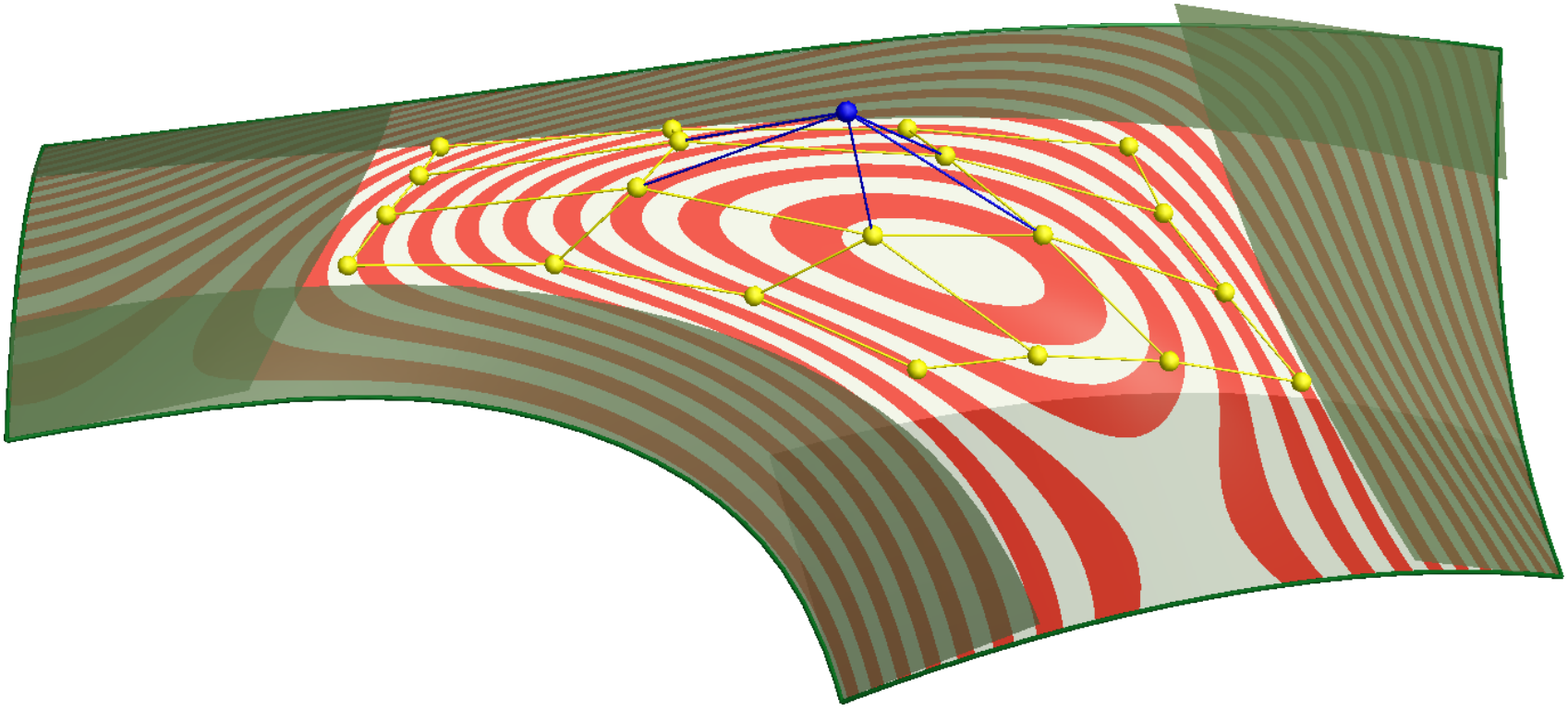




Example (isophote lines)



Example (isophote lines)



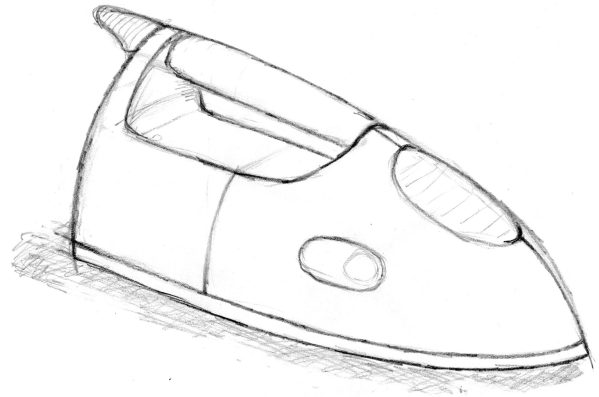
Outline

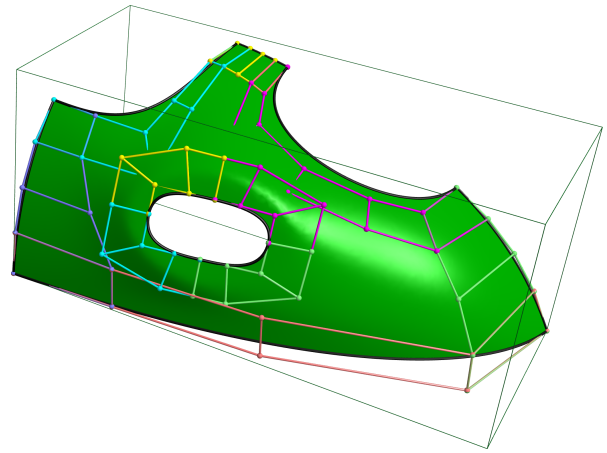
Conclusion

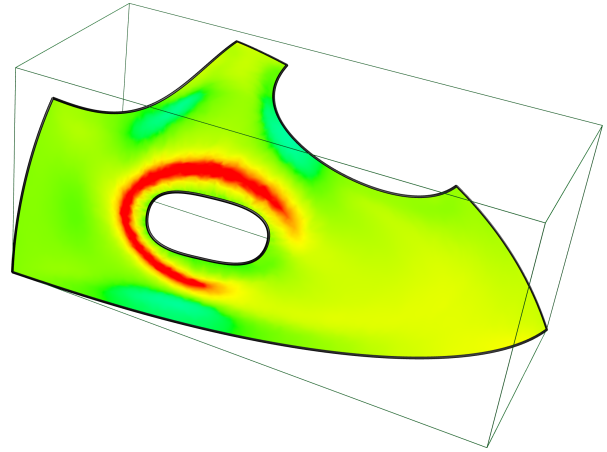
1. Transfinite interpolation surfaces
   - Katō's patch
   - Charrot–Gregory patch
   - **Midpoint patch**
2. Control-point–based surfaces
   - S-patch
   - Generalized Bézier patch
   - **Generalized B-spline patch**
3. **Hybrid patch**
   - Combines GB with Katō's patch
   - Interpolation of arbitrary boundaries
   - Natural control over the interior

Limitations:

- Not CAD-compatible
- Cannot handle *extreme configurations*







Related papers

1. Midpoint patch:

   P. Salvi, T. Várady, *Multi-sided surfaces with fullness control*.
   GrafGeo Conference Proceedings, pp. 61–69, 2016.

2. Tansfinite interpolation with S-patches:

   P. Salvi, G^1 *hole filling with S-patches made easy*.
   KÉPAF Conference Proceedings, #1, 2019.

3. Generalized Bézier patch:

   T. Várady et al., *A Multi-sided Bézier patch with a simple control structure*.
   Computer Graphics Forum, Vol. 35(2), pp. 307–317, 2016.

4. Generalized B-spline patch:

   M. Vaitkus et al., *Multi-sided B-spline surfaces over curved, multi-connected domains*. **Computer Aided Geometric Design**, Vol. 89, #102019, 2021.

https://3dgeo.iit.bme.hu/