Natural control for multi-sided surfaces

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Outline

Motivation

Generalized Bézier patches
  Curved domain
  Generalized B-spline patches

Ribbon generation
  Cross-derivatives by parameters

Interior control
  Blending functions

Tools for editing
  Control vectors
  Proportional editing

Conclusion
Multi-sided surfaces
Genuine multi-sided parametric surface patches – A survey

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ABSTRACT

A state-of-the-art survey is presented on various formulations of multi-sided parametric surface patches, with a focus on methods that interpolate positional and cross-derivative information along boundaries.

Keywords:
Multi-sided surfaces
Ribbon-based surfaces
Transfinite interpolation
Control point patches
Surface modeling
General topology
Classification and constituents

Implicit surfaces

Parametric surfaces

Patchwork of triangles or quads

Genuine multi-sided

Other schemes

Blend of surface interpolants

Weighted control points

Side-based

Corner-based

Boolean sum

Side-based

Corner-based

Interconnected

Ribbon constraints

Patch equation

Domain

Parameterizations

Blending functions

Editing capabilities
Classification and constituents

Implicit surfaces
Patchwork of triangles or quads
Parametric surfaces
Genuine multi-sided
Other schemes

Blend of surface interpolants
Weighted control points
Side-based
Corner-based
Boolean sum
Interconnected

Ribbon constraints → Patch equation → Domain → Parameterizations → Blending functions → Editing capabilities
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Generalized Bézier patches

\[ S(u, v) = \sum_{i=1}^{n} \sum_{j=0}^{d} \sum_{k=0}^{(d-1) \div 2} \cdot C_{ijk} \mu_{i,j,k}(u, v) B_{i,j,k}^{d}(u, v) + C_{0} \cdot B_{0}(u, v) \]

\[ B_{i,j,k}^{d}(u, v) := B_{j}^{d}(s_{i}(u, v)) \cdot B_{k}^{d}(h_{i}(u, v)) \text{ with } (s_{i}, h_{i}) \text{ local parameters} \]
Curved domain
Multi-connected domains
Generalized B-spline patches
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Cross-derivative strength
Setting by local parameters
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Conclusion
‘Templates’ from medial axis – distance parameters
‘Templates’ from medial axis – parametric medial axis
‘Templates’ from medial axis – MAT-based quad structure
‘Templates’ from medial axis – $T_2$ skeleton
‘Templates’ from medial axis – $T3$ skeleton
‘Templates’ from medial axis – $T4$ skeleton
‘Templates’ from medial axis – $T5$ skeleton
Degree synchronization
Distributing weight deficiency proportionally
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Editing boundaries by control vectors – Exact $G^1$
Editing boundaries by control vectors – Approximate $G^1$
Editing the interior proportionally
Editing the interior proportionally
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Conclusion & future work

▶ Curved, multi-connected domains
  ▶ Handling of highly curved boundaries
  ▶ Natural cross-derivative lengths
  ▶ MAT → interior control structure
▶ Interior blends
  ▶ Proportional weight deficiency distribution
▶ Editing
  ▶ Boundary CPs → implicitly by control vectors
  ▶ Interior CPs → simultaneously with a falloff function

Next: interior control structure for generalized B-spline surfaces
Related papers

1. **Multi-sided patch survey**
   T. Váray, P. Salvi, M. Vaitkus:
   *Genuine multi-sided parametric surface patches – a survey.*

2. **Modeling with control vectors**
   P. Salvi, M. Vaitkus, T. Váray:
   *Constrained modeling of multi-sided patches.*

3. **Independent interior controls**
   P. Salvi:
   *Intuitive interior control for multi-sided patches with arbitrary boundaries.*

4. **MAT-based interior controls**
   M. Vaitkus, P. Salvi, T. Váray:
   *Interior control structure for Generalized Bézier patches over curved domains.*
   *Computers and Graphics*, 2024. (accepted for SMI’24)

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