Topology-preserving polyhedral design using ribbon-based multi-sided patches

[CAD 199:104110 (2026)]

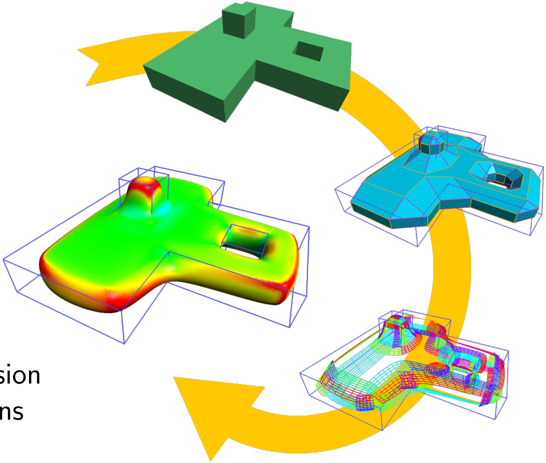
Péter Salvi[†], Jázmin Szörfi[‡], Márton Vaitkus[†], Tamás Várady[†]

[†]Budapest University of Technology and Economics
[‡]Shapr3D Ltd.

SPM'26

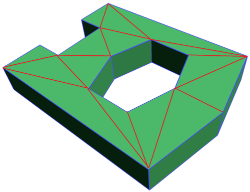
Istanbul, Jul 6–9, 2026

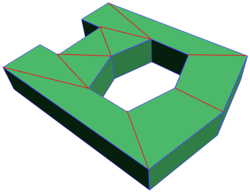
Outline

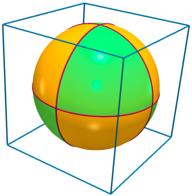


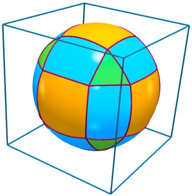
Polyhedral Design with Concave Faces & Holes

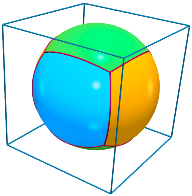
- Recursive subdivision
  - Catmull–Clark uses face centroids
  - Doo–Sabin shrinks faces
  - Concave faces need decomposition
    $\Rightarrow$ Ambiguous, high-valence vertices
- Direct surfacing methods
  - Curve network from the polyhedron
    - Dual (vertices $\leftrightarrow$ faces)
    - Doo–Sabin (vertices/edges $\rightarrow$ faces)
    - **Topology-preserving**
  - Also for curve network based design:
    - Initial network from polyhedra



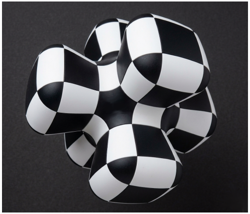




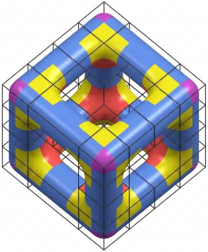




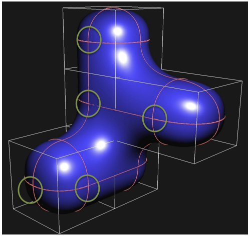
Related Work – Polyhedral Design with *n*-Patches



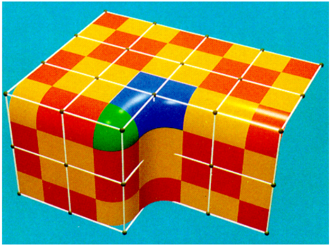
[Wang et al., 1999]



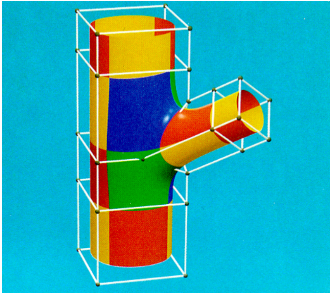
[Zheng et al., 2005]



[Rockwood et al., 2016]



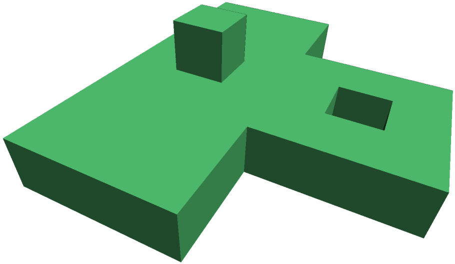
[Loop–DeRose, 1990]



Workflow

Computation steps:

1. **Input polyhedron**
2. Auxiliary polyhedron
3. Ribbons
4. Surfaces

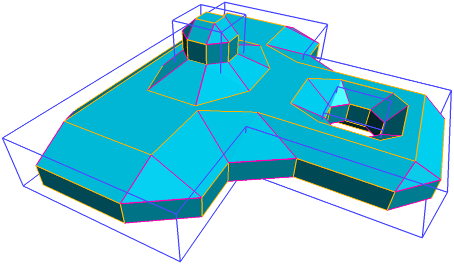


- Input can be concave, multi-sided & multiply connected

Workflow

Computation steps:

1. Input polyhedron
2. **Auxiliary polyhedron**
3. Ribbons
4. Surfaces

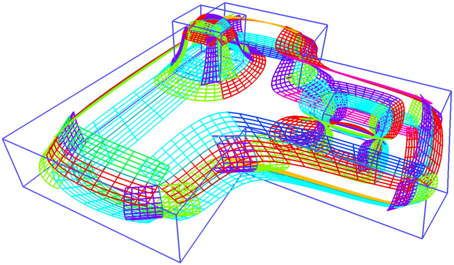


- Input can be concave, multi-sided & multiply connected
- Auxiliary created by a chamfering/offsetting procedure

Workflow

Computation steps:

1. Input polyhedron
2. Auxiliary polyhedron
3. **Ribbons**
4. Surfaces

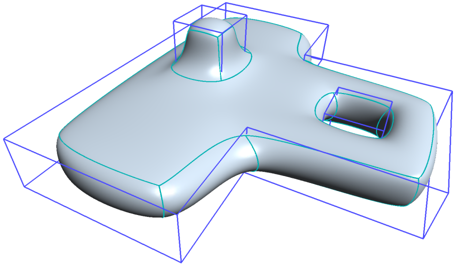


- Input can be concave, multi-sided & multiply connected
- Auxiliary created by a chamfering/offsetting procedure
- G^1-compatible ribbons directly from the auxiliary

Workflow

Computation steps:

1. Input polyhedron
2. Auxiliary polyhedron
3. Ribbons
4. **Surfaces**

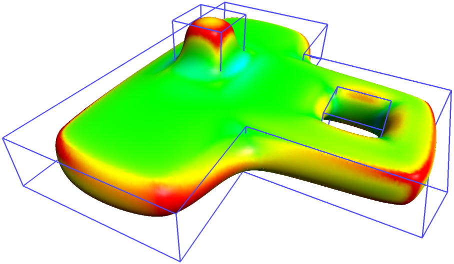


- Input can be concave, multi-sided & multiply connected
- Auxiliary created by a chamfering/offsetting procedure
- G^1-compatible ribbons directly from the auxiliary
- Patches interpolate the ribbons at the boundaries

Workflow

Computation steps:

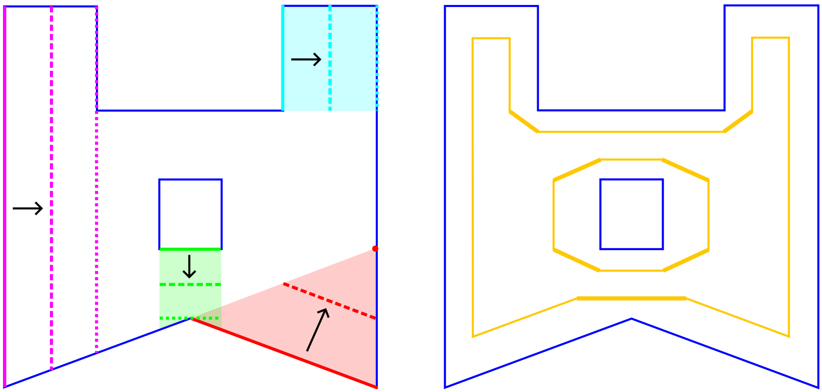
1. Input polyhedron
2. Auxiliary polyhedron
3. Ribbons
4. Surfaces



- Input can be concave, multi-sided & multiply connected
- Auxiliary created by a chamfering/offsetting procedure
- G^1-compatible ribbons directly from the auxiliary
- Patches interpolate the ribbons at the boundaries

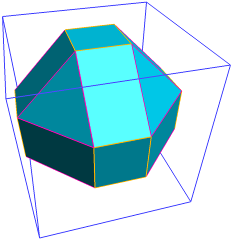
Let's see these steps in detail!

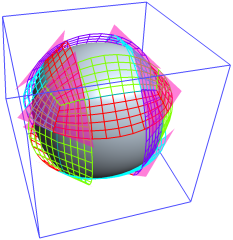
Auxiliary Polyhedron



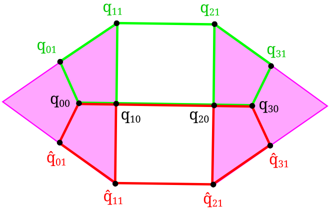
(+ some corrections for better patch quality. . .)

Ribbon Generation

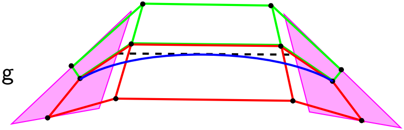


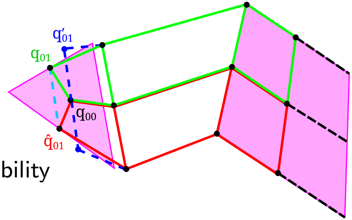


Preliminary ribbons by midpoints

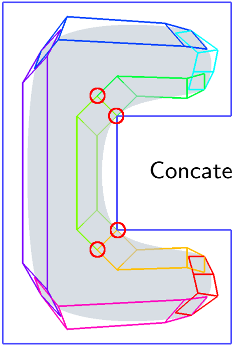


Scaling



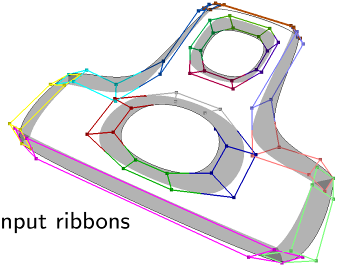


Compatibility

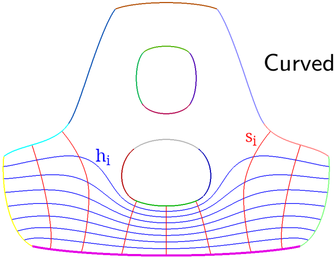


Concatenation

Patch Creation – Generalized B-Spline Surface (GBS)



Input ribbons

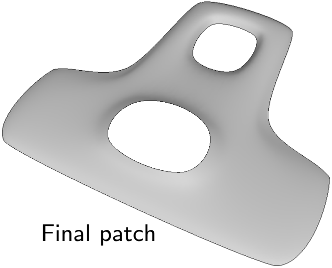


Curved domain

- Multi-sided, multiply connected
- Curved domain, local parameters
- Patch equation:

$$\mathbf{S}(u,v) = \sum_{i,j,k} \mathbf{C}_{ijk} \cdot \mu_{ij}(u,v) N_j(s_i) B_k(h_i)$$

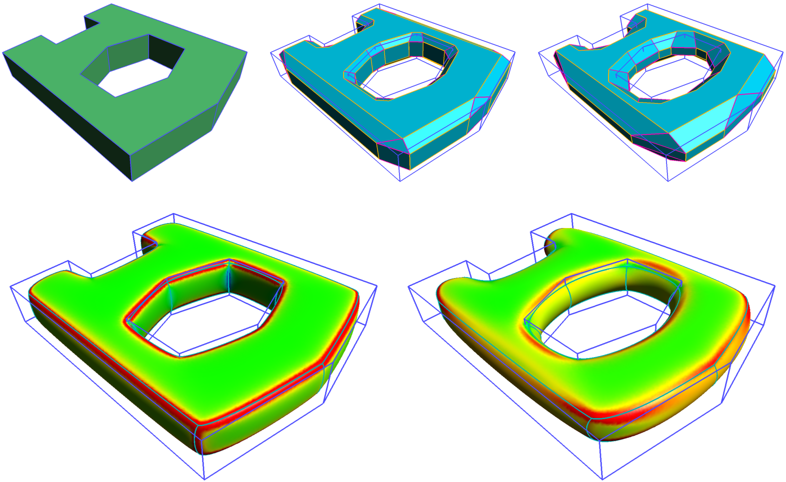
- Rationally scaled B-spline/Bézier



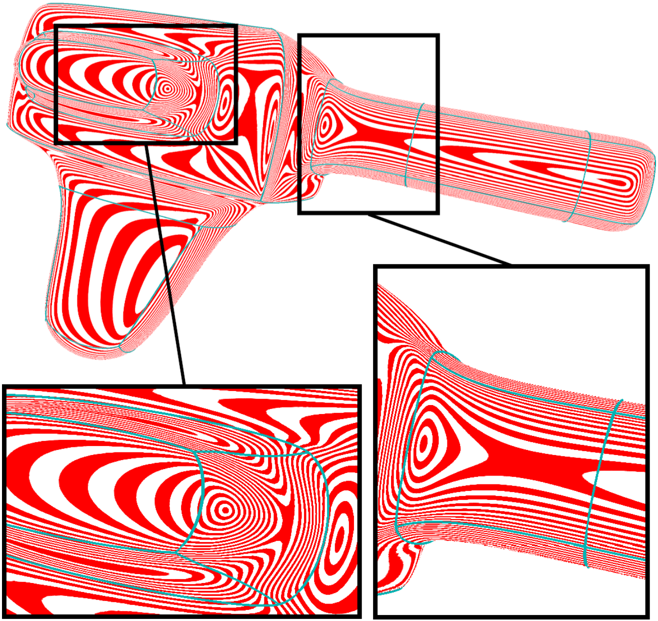
Final patch

Discussion

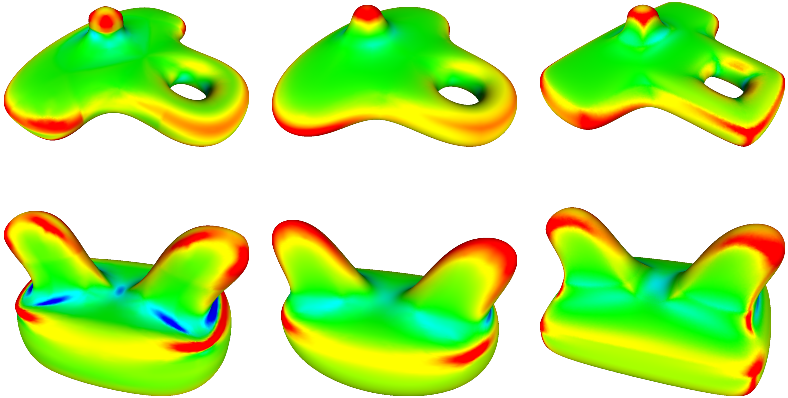
Fullness Control



Patch Quality

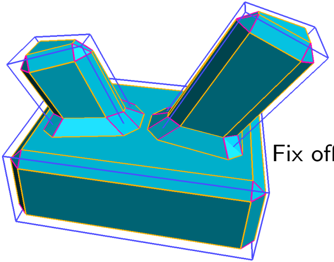


Comparison with Subdivision

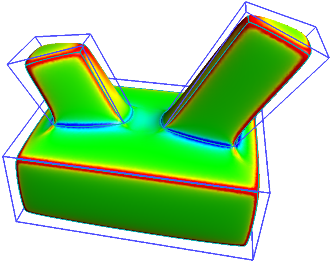


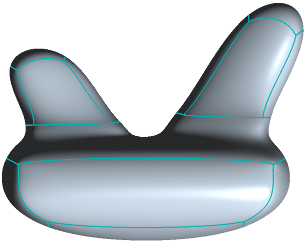
Doo–Sabin Catmull–Clark Our method

Alternative Chamfering

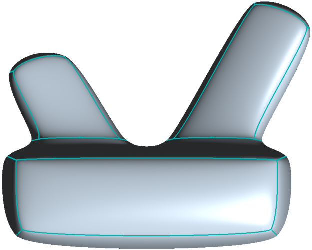


Fix offsets





Default offsets

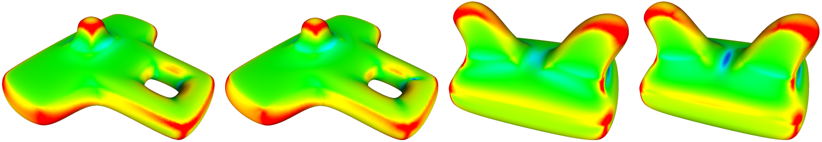


Edge-symmetric offsets

Alternative Patches

Other curved domain based patches can be used...

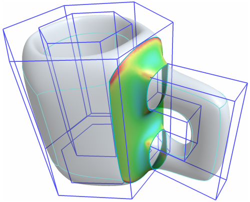
- Biharmonic patches [Vaitkus, 2023]
  - Minimize a discrete Laplacian (thin-plate energy)
  - Interpolate G^1 boundary conditions (from the ribbons)
  - Very similar results
- Kato patches? [Kato, 2000]
  - The basis of SuperD [Rockwood et al., 2016]
  - Can handle holes
  - Highly curved "concave" edges $\rightarrow$ curved domain?

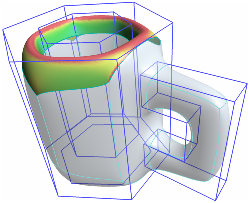


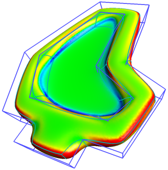
GBS | biharmonic | GBS | biharmonic

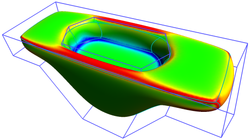
Conclusion & Future Work

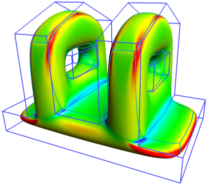
- Topology-preserving surfacing
- Handles n-sided, multiply connected faces
- Based on an offsetting process
- Uses Generalized B-Spline (GBS) patches
- Next:
  - Open control meshes
  - G^2 continuity
  - Optimized domain mapping etc.

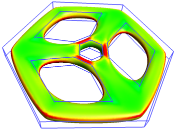












References (polyhedral design with multi-sided surfaces)

1990 Ch. Loop, T. D. DeRose: *Generalized B-spline surfaces of arbitrary topology*. **SIGGRAPH '90 Proceedings**, pp. 347–356.

1999 X-F. Wang, F-H. Cheng, B. A. Barsky: *Blending, smoothing and interpolation of irregular meshes using N-sided Varady patches*. **SMA '99 Proceedings**, pp. 212–222.

2005 J-J. Zheng, J-J. Zhang, H-J. Zhou, L-G. Shen: *Smooth spline surface generation over meshes of irregular topology*. **The Visual Computer**, Vol. 21, pp. 858–864.

2016 A. Rockwood, K. Gao, M. Farrell: *SuperD: SubD modeling without subdivision*. **SIGGRAPH '16 Talks**, #8.

2024 P. Salvi: *Polyhedral design with blended n-sided interpolants*. **11th GrafGeo Proceedings**, pp. 46–51. arXiv: #2601.19322.

2026 P. Salvi, J. Szörfi, M. Vaitkus, T. Várady: *Topology-preserving polyhedral design using ribbon-based multi-sided patches*. **Computer-Aided Design**, Vol. 199, #104110.

https://3dgeo.iit.bme.hu/