Surprising Deals

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You sit down at a bridge table, pick up your cards, and see the following hand:

$$\heartsuit\text{AKQJT98765432} \tag{1}$$

You are stunned. What are the odds of this?! After a cautious bidding sequence (intended to mislead the opponents), you bid—and of course lay down—the redoubled 7♡. Then, upon seeing the next deal, it is again difficult to keep a poker face:

$$\clubsuit\text{AK6} \quad \diamondsuit\text{AKQ3} \quad \heartsuit\text{AK4} \quad \spadesuit\text{AK8} \tag{2}$$

After a 2♣–2♢–4NT ping-pong, partner's 6NT bid promises 5–7 high-card points, and with at least 35 points between you, 7NT is no problem. This is your day.

But the cards have already been dealt again, and now you are greeted by a very different picture:

$$\clubsuit\text{AQ8643} \quad \diamondsuit\text{J75} \quad \heartsuit\text{Q64} \quad \spadesuit\text{3} \tag{3}$$

True, it contains a fairly good six-card suit, but unfortunately in clubs, so you cannot even open with it... however you look at it, with 9 HCP and a 6–3–3–1 distribution, this is a rather average hand, nothing special about it.

Or is there?

The probability of the three hands above is, of course, identical. From a 52-card deck, 13 cards can be chosen in $\binom{52}{13} = 635013559600$ different ways, so the probability of any specific hand is the reciprocal of this, about $1.6 \cdot 10^{-10}\%$.

Why, then, do we feel that the first hand is much less likely than the third? The reason lies in our conceptual framework: it depends on the "language" we use to talk about deals. The first hand is easy to describe in the language of bridge: "a hand with all hearts." The second can be captured in several ways, but perhaps the simplest is: "all aces and kings, plus the diamond queen," and then we list the remaining four small cards. In the third case, however, we have no particularly efficient way to describe the hand; we cannot do much better than listing the cards. The structure we perceive in a deal is what makes a hand seem more special or surprising.

This measure is called *Kolmogorov complexity*, defined as the length of the shortest description in a given (natural or programming) language. However, this depends heavily on the language.

Let us arrange the deck in natural order:

$$\clubsuit 2, \clubsuit 3, \ldots, \clubsuit \mathrm{K}, \clubsuit \mathrm{A}, \diamondsuit 2, \ldots, \diamondsuit \mathrm{A}, \heartsuit 2, \ldots, \spadesuit 2, \ldots, \spadesuit \mathrm{A} \tag{4}$$

Now let us look at which positions in this sequence correspond to the cards in the third hand:

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41 \tag{5}$$

These are exactly the first 13 prime numbers! With this knowledge—or in other words, in this new descriptive language—the hand suddenly appears much more surprising.

There is another source of the feeling of surprise. In bridge we always talk about *classes* of hands, never about specific hands. These classes are numerous and overlapping: they may be opening hands, balanced hands, two-suiters, etc. For the second example, "all aces and kings plus the diamond queen" defines a (quite specific) class; in contrast, "9 HCP and 6–3–3–1 distribution" is rather broad. Here it makes sense to say that one is less likely than the other, since the first description corresponds to at most $\binom{43}{4} = 123410$ hands (at most, because if it contained another queen, we would probably not describe it this way), while the latter corresponds to many times more (just the hands containing $\clubsuit$AKQ number $\binom{9}{3}^3 \cdot 9 = 5334336$).

When such a class appears, we can speak of the *information content* or *surprisal* of the event, defined as the negative logarithm of its probability (i.e., the logarithm of the reciprocal probability). For example:

$$I(\text{all hearts}) = \ln \binom{52}{13} \approx 27, \tag{6}$$

$$I(\text{aces, kings, and diamond queen}) = \ln \frac{635013559600}{123410} \approx 15, \tag{7}$$

$$I(13 \text{ HCP}) = \ln \frac{635013559600}{43906944752} \approx 3. \tag{8}$$

To summarize, we have seen two ways to quantify our sense of surprise, which can be combined: when classifying an event (a hand), we feel it is more special when (i) the class can be described more concisely, and (ii) the class has a smaller probability of occurring (i.e., greater surprisal). Both parts are necessary, because for example:

- The class "13 HCP" is concise, but its information content is small.
- If the "class" is simply the explicit listing of the cards, then the information content is maximal, but the description is not concise.

The choice of descriptive language (and thus the classes it can express) is part of the notion of surprise: a paradigm shift can re-evaluate an event, as we saw in the example of the third hand.